Using the van Hiele K-12 Geometry Learning Theory to Modify Engineering Mechanics Instruction

Janet M. Sharp
Montana State University - Bozeman

Loren W. Zachary
Iowa State University

Abstract

Engineering students use spatial thinking when examining diagrams or models to study structure design. It is expected that most engineering students have solidified spatial thinking skills during K-12 schooling. However, according to what we know about geometry learning and teaching, spatial thinking probably needs to be explicitly taught within the confines of engineering-specific contexts in college. The van Hiele theory of geometry learning explains geometry understanding as a series of more and more sophisticated ways to reason geometrically. The theory is known for its use in guiding K-12 geometry instruction. This paper describes the theory and explains how one engineering mechanics professor used it to re-conceptualize and re-structure his approach to teaching an engineering mechanics class. In particular, we describe his use of the van Hiele theory to move students toward success with free-body-diagrams, diagrams requiring complex spatial thinking and often a "point of departure" for most undergraduate engineering students.

Spatial Thinking as an Engineering Skill

Engineers are charged with the task of designing structures that make up our human-made environment. As such, they must effectively create, analyze, and interpret drawings and physical models, which serve as representations of the final structures to make predictions about how structures will act during use. These representations run the gamut from scaled, highly-detailed models, closely approximating the final structure to abstract (and often non-proportional) drawings or sketches, specifically drawn to isolate sections of the structure for analysis. For example, free body diagrams of a bolted joint are necessary for detail design in comparison to the shear and bending moment diagrams required to size and design a beam. All these efforts require strong spatial thinking skills on the part of the engineer. How do engineers become experts at developing this kind of thinking? The purpose of this paper is to outline a learning theory for spatial thinking and discuss the nature of geometry concept knowledge and then to describe how these played out in an engineering mechanics class, where students needed to exercise spatial thinking.

Both scaled and approximate representations have their places in engineering. But the mental tools necessary to (1) decide which kind of representation is needed for a given context and (2) isolate important elements of the representation for further study or analysis are often over-looked skills, because after a while, many engineers seem to do it naturally. During their undergraduate studies, engineering students need to be taught how to interpret engineering information and call up related mathematical equations inherent in a diagram before they can be successful in developing spatial thinking and corresponding engineering concepts. The ability to isolate vital information from a visual, representational context is an important engineering skill related to spatial thinking that budding young engineering students need to learn.

Although spatial thinking is a skill that can and should be taught even at the college level, it should not be taken lightly. Yakimanskaya (1991), a mathematics educator, nicely conveys the complexity of spatial thinking as a "form of mental activity which makes it possible to create spatial images and manipulate them in the course of solving various practical and theoretical problems" (p. 21). It should be noted that even when confronted with a physical model, an engineering student would still create a mental image of the model and manipulate it when thinking about resolving some problem situation with the structure. According to Yakimanskaya (1991), when people use spatial thinking to analyze and interpret representational figures, a constant mental re-negotiation occurs. This re-coding of mental images to identify and magnify important components represented in the situation is the development of spatial thinking.

Since a broad range of mental tools - like dexterity and flexibility with spatial thinking, particularly about drawings and physical models of structures - are heavily used in engineering mechanics, engineering professors must assume the responsibility of educating students to recognize and apply these tools in engineering-specific contexts. Consider contexts that might appear in a mechanics course. Classic problems like studying compression and tension using free-body diagrams to create, analyze, and predict results of forces acting on an object or figure require students to mentally organize information communicated in a specific figure and hence, think spatially.

As engineering professors work to build spatial thinking in their undergraduate students, it is important to keep in mind the K-12 experiences that those students have experienced. General spatial thinking, measurement, and analysis and interpretation of figures are part of the K-12 school mathematics of geometry (National Council of Teachers of Mathematics, 2000). Students have sorted shapes and made predictions about figures. After viewing Figure 1, high school geometry students

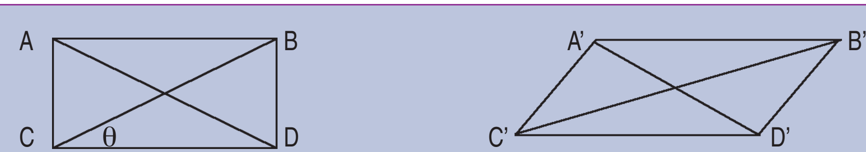

Figure 1. A rectangle and its deformation

might prove that the first shape is a rectangle, given only the information that the length of the diagonals are equal and that opposite sides are parallel ($\overline{AD} \cong \overline{BC}$ and $\overline{AB} \| \overline{CD}$ and $\overline{AC} \| \overline{BD}$)

College mechanics students would encounter the rectangle in a more dynamic fashion. Deformation of the original rectangle into a (non-rectangular) parallelogram, shown in the second part of Figure 1, is used to study and determine the strain along the $\overline{C'B'}$ diagonal by calculating the change in length of $\overline{C'B'}$ after it is deformed from the rectangle. In this process, students apply ideas from trigonometry to make a deduction about the nature of the strain on the deformed rectangle. In the end, they form a generalized representation of the strain along the diagonal (line segment $\overline{C'B'}$) at an angle _ with respect to the horizontal axis, segment $\overline{C'D'}$.

A college mechanics student learning to mentally manipulate and draw inferences from free-body diagrams to understand how and which stresses are created in a bridge have traveled a long way from typical K-12 geometric experiences. The successful engineering mechanics student integrates mathematics ideas into engineering knowledge and ceases to view that knowledge as mathematical because of the engineering context (Maull & Berry, 2000) whereas as a K-12 student, that same student would have used only general geometry knowledge. After all, the goals of K-12 education are much more general than the goals of engineering education. Nonetheless, the foundation of college students' experiences rests on geometry knowledge learned in school. Tapping into those K-12 spatial thinking experiences can be a useful way for mechanics professors to ground instruction that is so focused on engineering contexts while building on students' existing K-12 knowledge. Part of students' existing mathematics spatial thinking knowledge includes basic geometry concepts.

Concept Knowledge as an Engineering Skill

Although concept knowledge has proved difficult to satisfactorily define, for the purposes of this paper, we refer to Carpenter's (1986) description of a concept as a relationship between two ideas. A concept is also thought to be the rich set of these single mental relationships. Being able to flexibly access and use the information in the relationship means the concept is well-known. At the risk of over-simplifying the idea of concept, we work from the premise that a concept is a collection of these simple relationships. In geometry, then, pieces of a concept of *square* might be "all squares have four equal sides," "a four-sided shape is not necessarily a square," or even "squares do not have curved sides." The concept is not singularly in any one of the given statements; rather it is the collection and totality of the statements together with the student's facility with accessing that information at the correct time to solve a problem.

A context in a problem can be persuasive over and above concept knowledge. Mentally accessing concept information requires students remove themselves from the context. That is, to isolate the important information from the context of the problem and use it to solve the problem requires strong concept knowledge. When learners can generalize past a specific context, it can then be assumed they have developed sound concept knowledge. Carpenter (1986) is careful to explain that when adult learners have inadequate concept knowledge about a particular kind of situation, they are initially quite limited in their abilities to make sense of the situation. At this point, they are only able to consider exact replica representations of the problem situation and they create those representations based solely on the specific data given in the situation. It is only after adults have gained an elaborate network of concept knowledge, rich with meaningful relationships, that adult learners are able to build and create sophisticated, general representations of the situations that include relations not directly stated in the problem. When Maull and Berry (2000) found engineering students unable to extract important information from an engineering context, it was likely due to a lack of mathematical concept knowledge about the information.

Teaching Geometry (Spatial Thinking and Geometric Concepts)

Since a portion of what university engineering professors teach is related to geometry, it would seem that geometry learning theory could inform the engineering professor about how students would go about acquiring geometric-based engineering knowledge. A learning theory that has gained acceptance in the mathematics education community (Burger & Shaughnessy, 1986; Senk, 1989; Usiskin, 1982) is the van Hiele theory (Fuys, Geddes & Tischler, 1988; van Hiele, 1986). The theory hypothesizes five levels of understanding, through which students serially progress (Usiskin, 1982, Teppo, 1991). Consistent with constructivism (Burger & Shaughnessy 1986), this theory holds that learning builds upon or rearranges existing knowledge and can be developed with appropriate teacher questioning (Crowley, 1987). In this paper, we focus on the first three levels of the van Hiele theory of geometry thinking.

Van Hiele's Geometry Learning Theory

Van Hiele's theory of geometry learning is sequential and each of the five levels can be characterized by several identifiable student actions. The theory does not relate to chronological ages (Teppo, 1991; Burger & Shaughnessy, 1986,) it does not matter that the engineering student is an adult. The stages apply for each spatial thinking idea engineering students must learn. Since the fourth and fifth levels deal with abstract geometric proofs, these levels are not presented here. (For a full treatment of van Hiele theory, including a description of the remaining two levels, read van Hiele, 1986 and Fuys, Geddes, & Tischler, 1988.)

Level 0 – visualization. At this level, students view an object or figure through a global lens, not considering its individual components. First encounters with any spatial object should be interpreted according to "how they look," even for a college engineering student. So, engineering professors should first provide engineering students with opportunities to sort and classify figures according to visual differences. Students should simply make statements based on the entire visual image. Do these two figures look like trusses? Does this structure look like it would be stable?

Level 1 – Analysis. At this level, students remove themselves from global visualizations to study specific components of the image. They step close to the figure and notice characteristics and properties. They might make a list of all the things they notice about the figure. Is it rotating? Is it static? What sort of stresses are involved? It is important to note that the word, analysis, in the van Hiele sense, is different from common use of the word, which typically includes making conclusions or deductions. Before being able to make even informal deductions, which is described momentarily, students must be able to spatially think about and analyze embedded components of figures. When mechanics students attempt to analyze free-body diagrams, they will not succeed without first acquiring the skill to visualize.

Level 2 – Informal Deductions. At this level, students deduce some fact about the object or figure, as a consequence of something already known about it. Students cease to rely on visualization (level 0), a property list (level 1), or empirical evidence (also level 1). They use relationships to make a conclusion, stating consequences without offering proof. Much of mechanics is situated at level 2, where students create, interpret, and predict with free-body diagrams. Thinking spatially about free-body diagrams and interpreting these mental images requires students to logically interrelate previously discovered rules and properties. Such interpretation is characterized as level 2 thinking (Fuys, Geddes & Tischler, 1988).

Sequencing Teaching

Engineering students often struggle to use previously learned mechanics rules and properties to make deductive claims about how a theory plays out in engineering mechanics situations. Difficulties making sound deductions emerge because students have not fully engaged in analysis (level 1) thinking about the situation. It is also likely that this lack of analysis accompanies a decided lack of concept knowledge. So, before asking engineering students to make informal deductions, they should be encouraged to make analysis-level statements about the specific situation before moving to deduction-level statements.

K-12 research shows that when instructors teach at a level different from the level of their students, teachers and students literally do not understand each other (Teppo, 1991; Senk, 1989). For instance, consider the teacher who has displayed a square to a group of children and asked them, "Name the shape and tell why you know the name of it." It is not uncommon for young children to say, "Square, because it looks like one." Children in this situation are giving good reasoning, they just happen to be at level 0. The critical point of this learning opportunity rests with what the teacher does next. If the teacher does not expect such an answer, he/she might ask an inappropriate question, such as, "Why?" (again) or "Look at the sides, what do you notice?" In the children's minds, they have already answered the first question and they don't know what the teacher means by his/her second question about the sides. To them, the sides are not different from the square, what is there to look at? They do not understand what the teacher wants from them. A better next step would be to cover up the unimportant pieces of the picture and ask, "What do you see now?" If no progress is made, more examples are needed. Seeing general trends in a collection of examples is level 1 reasoning. Several pictures of several kinds of squares should be provided so young children have the opportunity to create a more general image.

To clarify, an engineering example is shown in Figure 2. The professor says, "For the pinned truss, characterize the internal forces in each of the members." A student response of "It looks like the whole thing is in compression." Although incorrect, it is an example of a visualization-type answer (level 0) to the directions. By looking at pin A and using the method of joints, it eventually becomes clear that the member is a zero force member. But looking at parts of the diagram, such as the vertical member or horizontal member, requires the student to think

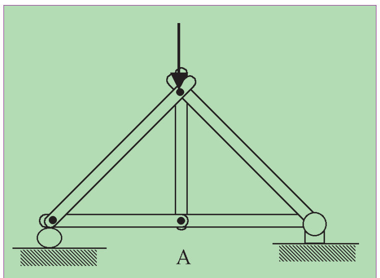

Figure 2 – A zero-force member example

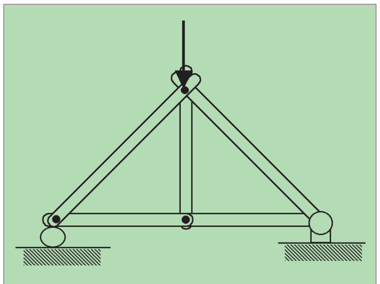

Figure 3 – Best example of a pinned truss

Essential Features:
- All joints are pinned and free to rotate.
- Members are only attached at two locations by pins.
- Forces are applied at the pins

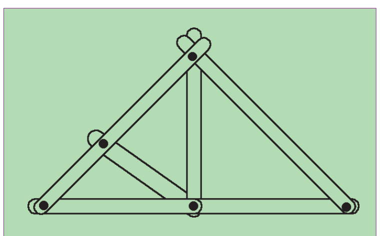

Figure 4 – A non-example

- A member has been added that creates a third pin in a member.
- This addition makes it a frame.

at the next van Hiele level (level 1), which is analysis. The instructor could literally cover most of the figure to allow a student to focus on a vertical member.

Educational research supports the notion that all learners pass sequentially through van Hiele levels for any geometry idea learned and that any given learner can be at different levels for different concepts (Burger & Shaughnessy, 1986; Teppo, 1991). The job of the instructor is simply to determine the level of thinking of the student and then modify instruction to match that level. (We say, "simply," not to imply this is a simple process, but to imply that it is a straightforward process, with this being the only place a teacher and his/her students can begin.) Instructors must also monitor the levels of thinking exhibited by students because "students may move back and forth between levels quite a few times while they are in transition from one level to the next" (Burger & Shaughnessy, 1986, p. 45). The instructor who is aware of this sequence and of strategies to address the disparity in thinking levels can ensure that spatial thinking develops (Crowley, 1987).

How does one monitor students' thinking levels? By listening to students' answers! If students reason that a figure will act a certain way because it "looks" like it should, they are giving a level 0 answer. Students should be shown how to isolate (analysis level) the important elements of the figure and to compare two or more figures. Professors can ask focusing questions to help students recognize the important components. Ask the student to look at a specific member in the figure and to explain what its role is in the figure. Another way to help students focus on the important pieces of the figure is to literally hide a portion of the figure and ask students what would have to be under the cover for the structure to be stable. Such teaching strategies help students move through the analysis level. Then, when students are asked to predict what parts of the figure act in what ways, students can give informal deduction type answers when studying information in Figure 2. They might say, "Well, if this member is in a pinned truss, then I know only an internal axial force exists in the member."

It may seem that a typical college engineering student should have experienced success in high school geometry and thus, would have moved into the upper levels of van Hiele thinking. Burger & Shaughnessy (1986) found that students regress after finishing high school geometry. Without constant effort to maintain a high van Hiele level, students do not retain informal deductive thinking. Engineering mechanics college students may still need to experience sequential instruction in order to tackle geometry-based tasks.

The art of appropriately sequencing geometry instruction also must employ theory related to the development of concept knowledge. Of most critical value is the use of a wide variety of examples *and* non-examples when students are thinking about conceptual ideas (Carpenter, 1986; Fuys & Liebov, 1997). See Figure 3 for an example and Figure 4 for a non-example of a pinned truss. Specifically designing learning experiences to include non-examples enables students to further isolate important pieces of a concept and solidify relationships that organize the concept. Moreover, students must be shown a *collection* of example and non-example images if they are to develop a rich collection of relationships making up the concept understanding about the idea in question. Fuys and Liebov (1997) explain the importance of "best" examples and "concept cards" in concept-based lessons. Best examples are simple examples of the idea and communicate *only* essential features of the concept, without extraneous or confusing information. (See Figure 3.) Often, real-world examples of concepts contain more than essential features and, although they serve as good examples, they do not help students isolate and focus on the concept in question. Hence, they are not *best* examples. In addition, since concept knowledge manifests itself in an ability to generalize the concept away from a given context (Carpenter, 1986) it is imperative that nonessential information be minimized. Isolating and pulling only vital information from a diagram is particularly important for engineering students, since Maull and Berry (2000) found engineering students prefer verbal descriptions of mathematical ideas but diagrams for mechanics. Abilities to appropriately use a diagram are therefore important.

Concept cards are teaching tools that help students refine and organize their concept knowledge by showing what is expected. Concept cards are literally cards students study that have information designed to move them through the process of formulating concepts. These cards ask students to examine both examples and non-examples, classify provided specimens and finally, state a general description of the concept. In Figure 5, we complete these remaining elements of a concept card and simultaneously demonstrate how pictures can grow increasingly abstract. Notice how the last two images do not show members of the truss "looking like" boards and pins.

Applications to Engineering Education

So, what does this mean? Engineering professors must recognize that their courses involve some modicum of spatial thinking, that the thinking must

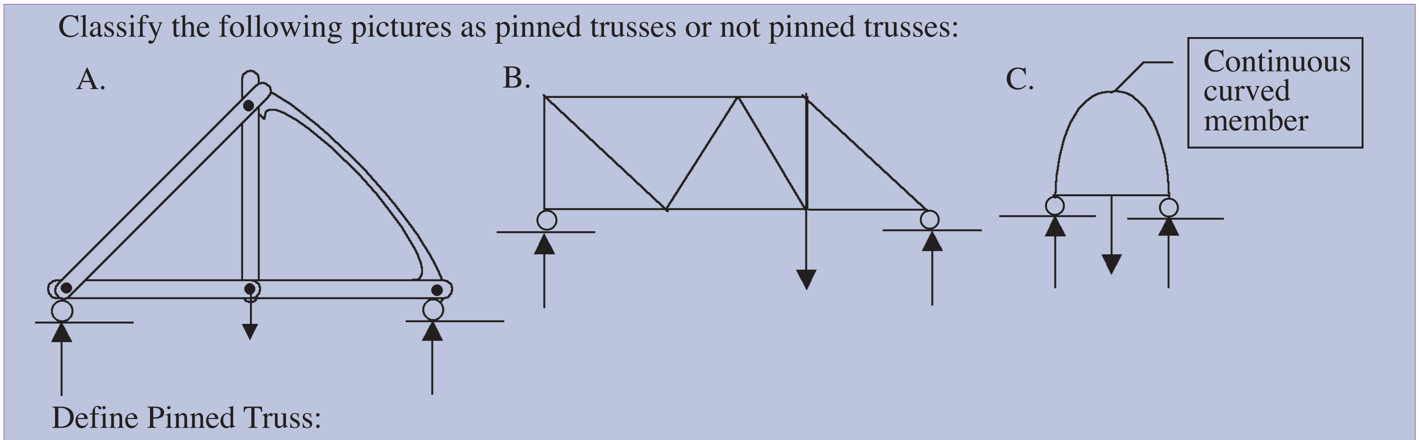

Figure 5 - The last two pieces of a concept card for a pinned truss

be consciously re-coded from non-engineering contexts into engineering contexts, and that students must use spatial thinking to understand engineering concepts. It is a sophisticated use of spatial thinking to focus on only characteristics of the figure that need to be addressed (Yakimanskaya, 1991) and a somewhat complicated process to interpret a figure. Being able to isolate important forces and to predict consequences of forces means coordinating concept knowledge with spatial thinking at van Hiele's informal deduction level.

Armed with knowledge about van Hiele and strategies for developing concept knowledge, mechanics professors can structure good questions, activities, and homework to identify and develop students' abilities to make informal deductions. For example, asking questions about similarities and differences between figures leads students toward analysis-level thinking. Without guidance, progression through these levels of thinking does not just miraculously happen. Professors must pay close attention to the levels of student explanations about the consequential mechanics activity of a structure while explicitly requiring them to use and refine their spatial thinking. Showing several visual examples and non-examples of real-world structures is critical at all van Hiele levels. For instance, students might encounter a figure that has an error related to the idea under discussion and be asked to identify the error. Or they might encounter unusual situations such as the pinned truss shown in Figure 3, which has a curved member, and be asked to analyze it before making an informal deduction, "The figure is a pinned truss and therefore each member including the curved member is a two force member."

An Engineering Mechanics Class

Here, we provide a case study of one undergraduate engineering class to show what transpired when an engineering mechanics professor utilized the van Hiele theory to develop one of his lessons. For a unit on stresses associated with combined loading, he developed a collection of 33 figures for his students to study across two class periods. A sample of these figures is shown in Figure 6. He assumed the students were not beyond van Hiele's level 1 thinking, so they were asked only to *analyze* the situation. "Look at these figures and decide whether or not any of the following internal forces and moments are present: T, M_y, M_z, V_y, V_z, P." Students were also specifically directed *not* to do any calculations, "Don't crunch any numbers," because executing computations would require them first to make informal deductions about the forces acting on the figures. In groups of four, students studied and discussed the figures for approximately 80% of the class period (40 minutes) and noted forces, moments, and equilibrium but did not indicate magnitudes. In fact, several of the figures did not have numbers, just directional activity. Additionally, students determined whether or not any of the internal forces and moments existed. Then, a whole class discussion ensued where groups of students demonstrated engineering concept knowledge (or lack thereof) as they reported their lists of properties for each figure. For example, for Figure 6, a student commented that she looked at "the level part first." Another student said, "I thought about the vertical part as if it were rubber." The professor followed the van Hiele theory by specifically directing them against making informal deductions about the figures. For example, students were expected to identify the existence of stress but not to speculate on how the stress was induced by the force. Predicting *how* stress is induced, rather than *identifying existence* of a stress, would have been requesting an informal deduction.

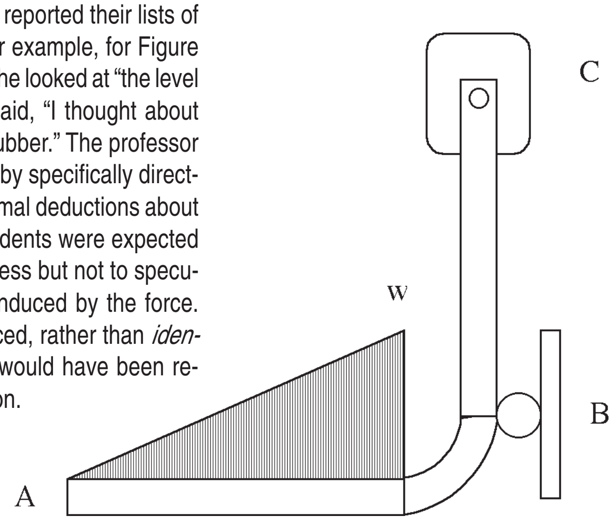

Figure 6.

At the beginning of the second class period, students individually studied a new figure with external forces and moments noted, without magnitude. They were directed, "Sketch the views of the problem to show what forces or moments exist at the indicated cross-section and which external load causes it." After six minutes, the professor engaged students in discussion, asking for agreement about properties at work in the figure. At one point, a student expressed concern about the existence of moment about Y in Figure 7. The professor asked students to draw a top view and pointed out features that would address the concern. Satisfied with their pictures, he drew for the students what he had expected them to sketch. "I want to show you how I would look at this." His instruction helped students analyze the figure because he modeled how an engineer would think spatially about the figure, calling out the specific components of the figure (analysis level) that needed their attention. Recall that being able to identify important characteristics is an important part of concept knowledge. By the middle of the second class period, several students asked questions that made it clear they were ready to make informal deductions about specific characteristics of figures, "Since this is a pinned truss, then wouldn't I need to calculate the load in this direction?" These students were making an informal deduction about the situation and were ready to move forward.

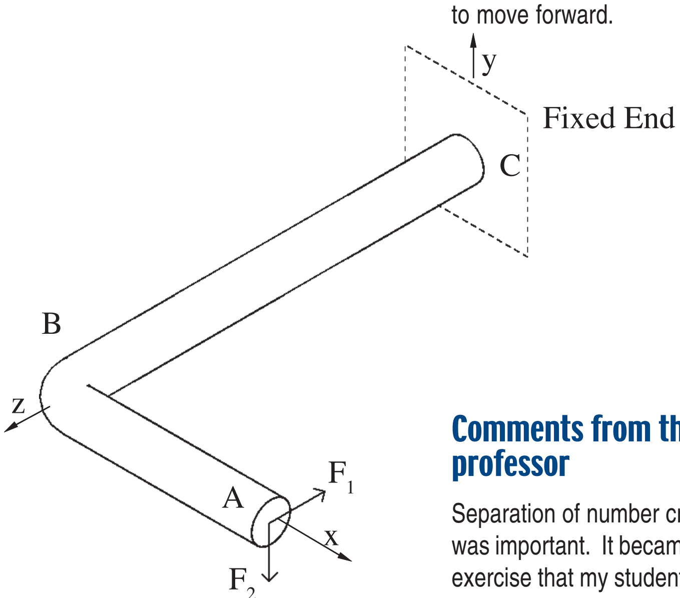

Figure 7.

Comments from the engineering professor

Separation of number crunching from visualization was important. It became evident to me during this exercise that my students needed more practice in visualization than I had realized. The students would also have benefited tremendously from a van Hiele approach when they learned how to use and draw free body diagrams in prerequisite courses. The concept card for the simple pinned truss problem was a good introduction for *me* in applying these principals. For engineering professors who wish to use these approaches, I suggest starting with a simple problem that requires both visualization and calculations but is still focused on visual information to hone your skills. It is easy to assume that using concepts to merely visualize (van Hiele level 1) is a low-level task, requiring minimal information extracted by the student. Be prepared to be surprised. I also teach an engineering course for elementary education majors, students who presumably have no more (and probably less) geometry concept knowledge than engineering majors. I found them as adept at learning pinned truss concepts, as my engineering majors, when I used the van Hiele theory and concept card approach. They could readily determine if a member was in tension, compression, or stress free. Although the approach may take a little longer in the beginning, I have found that as soon as students' responses indicate concepts are already mastered, then I can move forward quickly from there and make up any lost time.

Having students work in groups is important to allow them to dig deeper into the problems. By not allowing students to hide behind numbers and calculations, they are encouraged to talk about the concepts. In all fields of engineering, diagrams are used to understand how problems are constructed. The problem definition is an iterative process with assumptions and accompanying checks made on those assumptions. I have observed for years that students are not inclined to draw diagrams to help them solve problems. The process of visualizing problems on paper is apparently difficult to master without directed practice at doing so. Working in groups allows them to discuss their diagrams and to compare their thinking with their peers. The use of van Hiele as a basis for moving students to a better understanding of the problem definition and analysis is sorely needed in engineering education. What is easy for an expert to do quickly and accurately is, for the novice, a process of traveling down many unproductive paths in search of the answer. Although I believe traveling several paths teaches students to eliminate unproductive strategies and preserve productive ones, concept knowledge and their own drawings can help students more quickly recognize the productive paths. Helping students develop skill in this process, allows them to generalize approaches to problems that make sense to them. Allowing them to discuss their work with classmates encourages them to make their own sense of the situation and to defend their strategies.

Invitation to try the theory

When engineering students use spatial thinking to analyze and interpret models or figures, mental renegotiation of their images occurs, whether the engineering professor plans for it or not. It seems logical that engineering professors should therefore structure classroom activities and homework assignments with awareness of the geometry learning theory at work. In this way, they can expect students to experience more success at re-coding spatial images to identify and magnify important components represented in the situation in a way that is useful for engineering. As students develop informal deductive skills, analyzing and giving reasonable explanations about structures, they should be able to more correctly predict the outcome of various forces represented in the model or figure of, say, a bridge design.

It is likely that many engineering professors already structure courses, or portions of their courses, in this way, based on experiences with or intuition about good teaching. But this theory can be used to more fully understand and explain students' answers, and therefore, students' thinking. The next time a student says, "It just looks like it should work," the professor will know that the student is providing a level 0 answer and needs to be asked to focus on individual components of the diagram, making statements about potentially unique properties of the figure or model. Only after that will the student be ready to make useful informal deductions about what will happen to the figure and hence, the object. Through careful instruction based on the van Hiele theory, engineering mechanics instructors can recognize various levels of geometry thinking and plan subsequent instruction to build up students' spatial thinking and concept knowledge needed to solve engineering problems.

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Loren Zachary is professor of Aerospace Engineering and assistant dean for undergraduate programs in the College of Engineering at Iowa State University. In addition to teaching undergraduate mechanics courses, he works closely with K-12 teachers to support their efforts to use engineering contexts to make mathematics and science meaningful.

Janet Sharp is associate professor of mathematics education at Montana State University. She teaches graduate and undergraduate courses in mathematics and mathematics education. She studies strategies for helping future and practicing teachers understand the nature of teaching and learning in K-12 mathematics.