

Quantifying Aluminum Crystal Size Part 1: The Model-Eliciting Activity

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Introduction

We as engineering educators have traditionally approached teaching math, science, technology, and engineering concepts in a manner where we present content, enable students to develop discrete skills, and have the students apply the content and skills to the solution of a problem that is devoid of context. It is a comfortable and manageable pedagogy. We have the knowledge, we pass on the knowledge, and students apply the knowledge in neat and tidy ways. This mode of educating, however, implicitly assumes that students bring little to no knowledge of their own to a problem solving situation and our role is to inch them forward in easily assessable ways until they are deemed ready for messy, complex problems. So, the professor is the center of the educational experience, responsible for passing on knowledge and testing the accumulation of that knowledge by students.

At the other extreme in engineering, we assign very open-ended problems where now we expect students to bring to bear their combined knowledge to design a reasonably sound engineering solution. For first and second year engineering students, such problems can be overwhelming and outside of their educational experience. For the faculty creating and assigning the problems, anticipating what students will do or not do with an open-ended problem is problematic and often leads us to revising the problem on the fly in such ways as to lead the students down our preferred solution path or reduce the complexity of the problem – putting us again at the center of the educational experience.

This dichotomy led us to ask: “How can we enable students to use their own knowledge to construct new knowledge – essentially making learning more student-centered?”; “How can we help students transition from solving close-ended problems to very open-ended, realistic problems?”; “How can we construct open-ended, realistic problems that work and work well?”

To address these questions, we have adopted the models and modeling perspective developed by Lesh, et al. [1] to create model-eliciting activities (MEAs) that are engineering-based. The development, implementation, and assessment of these activities requires a new way of thinking about teaching. For student-centered learning to occur, the instructor must hold the belief that students do have knowledge and skills that they can draw on in realistic, complex problem solving situations. Certainly, a keen awareness of the nature of students’ knowledge, skills, life experiences, interests, and academic needs helps ensure that the problem context is accessible to all students. To encourage students to use their own knowledge and skills to develop unique solutions, the instructor must be willing to take on a different role in the classroom - facilitator. This is difficult – we are conditioned by our own educational experiences and teaching practice to guide students down a “true” engineering solution path when they are lost or straying. The instructor as facilitator needs to accept alternative ways of solving the problem. Part of the engineering students’ learning includes the evaluation of alternative solutions within the constraints of the context. Of course, there are degrees of correctness to a solution that must be taken into account, but within that constraint, the idea is to motivate students’ different ways of thinking about a given problem and empower them to solve problems they might perceive as difficult but worthwhile.

Unlike traditional in-class instructional methods, where the “teaching” is viewed as that which occurs in the classroom, most of the “teaching” occurs during the design and planning of the model-eliciting activity. This includes selection of the context and concepts for the problem, development of the problem to align with course objectives, and checking that the problem adheres to the guiding principles (see below). In our case, as we have been learning how to adapt the models and modeling perspective for engineering education purposes,

Abstract

Model-eliciting activities (MEA), specially designed client-drive, open-ended problems, have been implemented in a first-year engineering course and in secondary schools. The educational goals and settings are different, but the design of an MEA enables it to be versatile. This paper will introduce the reader to the principles that guide MEA design and demonstrate the flexibility of implementation by describing the use of an MEA, Aluminum Crystal Size, in these two educational settings.

model-eliciting activities, and subsequent model-development sequences [2], were written by teams of graduate students from engineering and mathematics education under the guidance of engineering faculty who teach a first-year engineering course. This mode of development ensured that a wide variety of engineering problem contexts were brought to the table, that we followed the guiding principles, and that we did not lose sight of the customers – the faculty teaching the courses in which these problems are implemented and the students who will be solving the problems.

This paper describes the purpose and intent of MEAs and details the six principles that guide their development. The paper will demonstrate the versatility of MEAs in terms of their ability to aid instructors in meeting multiple and varied educational objectives and their ability to be used in diverse educational settings.

Model-Eliciting Activities & Their Design Principles

From an engineering education perspective, modeling activities are open-ended, real-world, client-driven problems that require the creation or adaptation of a mathematical model for a given situation. The type of mathematical modeling activities described here are different from the open-ended problems that are typically assigned to students in their formal engineering education because the activities are specifically designed to prompt a *process* for solving problems, in addition to a specific *product*. While a strong product orientation is important to the success of students' in the engineering workplace, first-year students often have the idea that the "right" product is the first solution that they can quickly identify and implement to a personally satisfying level. Beginning engineering students often do not employ a problem solving process in which they are required to evaluate their trial solutions against the client's needs and engage in cycles of testing and revising their solution to the problem. The activities described here are designed so that teams of students need to express, test and revise their mathematical models to produce an acceptable product for a client. Requiring that students engage in a problem-solving process in addition to requiring that they create a product helps the development of higher-order thinking and reasoning and also counter-balances students' propensity to circumvent the problem solving process rather than grapple with the complexity of the situation. In engineering education, when

students externalize their solution processes (in their dialogue while engaged in small group work or in any required reports of intermediate solutions), the instructor gains opportunities to see and hear the processes and skills students bring to bear on the problem situation – all of which can inform instructional interventions that are immediately applied or archived for future use in subsequent classes.

MEAs are designed to be both thought-revealing and model-eliciting [1] and require students to mathematize (e.g., quantify, organize, dimensionalize) information in the context. Six principles for designing model-eliciting activities [1] have been adapted to the development of mathematical modeling activities for engineering courses, with some modification and refinement for the engineering context. We have found that the development of activities that meet all six principles are the most effective in providing problem statements to students that work in classroom practice. The six principles, as adapted to engineering contexts, are described here with reference to the Aluminum Crystal Size MEA (Appendix). In this activity, students in teams of four are asked to create a procedure to quantify crystal size using micrographs of various samples of metal.

1. *The model construction principle* means that the activity developed requires the problem solver to create a mathematical system to reasonably address the needs and purpose of a given client. The mathematical model can be a procedure, an explanation, or representation. A mathematical model, as used here, is a system that is used to describe another system, to think about a system, to make sense of a system, to explain a system, or to make predictions about a system. A mathematical model is a system that consists of (1) elements, (2) relationships among elements, (3) operations that describe how the elements interact, and (4) patterns or rules that apply to the relationships and operations (such as symmetry, commutativity, or transitivity). In the Aluminum Crystal Size MEA (ACS MEA), students are to develop a procedure for quantifying crystal size from micrographs by accounting for the different scale in each micrograph, the varied shapes and sizes of the crystals in each sample, and sampling of the crystals to be used in the measurement.
2. *The reality principle* implies that the question posed in the activity is a *realistic* engineering situation that requires the use of a mathematical model to solve the problem. We say "realistic" since students are in the

earliest phases of their engineering education, some problems need to have some of the layers of complexity removed to make the problem accessible to students. However, the goal is to ask, "Could this really be a question posed to engineers?" Rather than provide the students with decontextualized, scaffolded problems (i.e., problems that lead students to a particular predetermined approach), the model-eliciting activities are designed so that students create their own approaches to dealing with a realistic engineering situation. The situation should also be realistic in the sense that realistic assumptions can be used by the students to assess the quality of their solutions. By having students work in teams, bringing different types of skills to bear on the problem situation and collaboratively creating approaches to the problem solution, first-year engineering students are able to grapple with quite complex, realistic, engineering situations. In the ACS MEA, the reality principle is met by couching the problem in a context in which an aluminum manufacturer (the client) needs a method to quantifying crystal size for the purpose of developing a quality control system. In addition, the reality principle is met by the fact that material science engineers actually use micrographs to make such measurements [2].

3. *The generalizability principle* poses the question: "Does the model developed provide a way of approaching the situations that is sharable, transportable, easily modifiable and/or reusable?" In engineering, it is important that the tools designed for solving problems are useful beyond the team who developed them and applicable to situations that go beyond the immediate context. Consider computer tools used by engineers (e.g., spreadsheet software) and the mathematical models underlying these tools, the procedures embedded in them and the procedures that can be derived from them, vary greatly in their generalizability. Some are highly restricted to the peculiarities of particular problem situations, but others are taken out of their initial setting and applied to a wide variety of structurally similar situations. Some mathematical models, or tools, are designed to be reusable, and some are not. The greater benefit to industry comes from the potential for transportability. Some of the ways that model-eliciting activities can be designed to capture sharability are to develop tools that are intended to be used by others and work when new data are substituted. The aluminum manufacturer in

the ACS MEA needs to use the procedure developed by the students on unknown micrographs. The implication is that the procedure developed by the students must be useful to the client, not just the students working with the three provided micrographs.

4. *The self-assessment principle* means that the problem provides context and information needed to help students evaluate their progress as they work on a problem. In realistic situations, if engineers recognize the need for a given construction, description, explanation, then an explosion of ideas is likely to occur as a team begins to work on a problem. For students' proposed models to evolve over iterations to better models, they need to have ways to make informed selections, refinements, and elaborations. Thus, the self-assessment principle asks: "Does the problem statement suggest strongly appropriate criteria for assessing the usefulness of alternative solutions? Is the purpose clear (what, when, why, where, and for whom)? Are students able to judge for themselves when their responses need to be improved, or when they need to be refined or extended for a given purpose? Will students know when they have finished based on their personal knowledge and the information provided with the task?" In the ACS MEA, the students are provided with three micrographs on which to test their procedure. Such testing of their procedure on the micrographs should lead to cycles of expressing, testing and revising of their procedure.

5. *The construct documentation principle* means that the activity is not only model-eliciting, but thought revealing. Students' mathematical approach to the problem is revealed in the product they produce for the client. The construct documentation principle poses the question: "Will the students' response to the problem statement require them to explicitly reveal how they were thinking about the situation at the conclusion of their problem solving session?" The construct documentation principle contributes simultaneously to both learning and the documentation of learning. The activity is designed to naturally prompt students to externalize their current way of thinking by recording intermediate steps and solutions. This documentation process presents an opportunity for students to look back at their own progress, assess the evolution of the mathematical model they are designing, and think about the model as an object for re-

flection. Further, the construct documentation principle provides teachers and researchers with a window into students' ways of thinking, which in turn can inform subsequent instruction with the given set of students or in future course revision. In the ASC MEA, students were asked to develop their procedure by communicating with their team members on a discussion board provided through WebCT, a course management system. This provides the instructor with a high level of detail with regards to the processes students' use to solve the problem but can be overwhelming if the class is large. A more typical document produced by student teams is a memo to the client describing the procedure and its application to the three given micrographs.

6. Finally, the *effective prototype* principle means that the students' solution to a problem provides a useful prototype, or metaphor, for interpreting other situations. Long after the problem has been solved, will students think back on the given problem when they encounter other, structurally similar situations? To meet the effective prototype principle, the problem statement must create the need for a significant construct that will have use in other contexts. For the ACS MEA, the quantifying of the average size of objects from images has many applications beyond this context, including detection of grasses and broad leaf plants in an agricultural field, and estimation of roughness at the nanoscale [3,4].

These principles provide checks for the successful development of a MEA. Such guidance for writing open-ended problems is lacking in the literature, though open-ended problems or challenges provide the basis for similar pedagogies. It is worth noting at this point, the similarities and differences between MEAs and three other learning environments that use open-ended problems. Consider the How People Learn (HPL) framework [5] which maintains that effective learning environments have four dimensions: (1) knowledge centeredness, (2) learner centeredness, (3) assessment centeredness, and (4) community centeredness. MEAs embody these principles by (1) requiring students apply domain knowledge to the solution of the client's problem, (2) expecting students not only draw on the background knowledge that they bring to the learning environment but also create knowledge as they solve MEAs, (3) providing cues to encourage self-assessment of solutions, and (4) having the students work in teams so as to leverage their peers as a learn-

ing resource, respectively. The implementation of an MEA somewhat follows the Legacy framework [6] in that "The Challenge" is presented, students "Generate Ideas" on how to solve the problem taking into consideration "Multiple Perspectives" of the team members, the students "Research & Revise" and "Test [Their] Mettle" through model development and self-assessment, and "Go Public" in their response to the client. One difference between the Legacy Cycle and the implementation of an MEA lies in the potential need for students to research information external to the problem statement. While there is potential for including this aspect in MEAs, one of the advantages is that the activities *can* be designed to be contained within designated lab time, allowing instructors a means of handling course constraints that often accompany the inclusion of open-ended problems.

MEAs and their implementation also adhere to many of the essentials of Problem-Based Learning [7], where the learning environment drives the learning. MEAs are ill-structured, in that there exist multiple reasonable models that can meet the constraints of the task. MEAs are less dependent on inquiry where students pose a hypothesis, test it out, and revise based on what is learned. Rather, inquiry in MEA's takes the form of expressing a potential solution, or direction, testing it on peers and the constraints laid out in the problem, and then revising (or rejecting) the trial solution. This is more typically thought of as a design process. Other similarities to Problem Based Learning (PBL) include the fact that the context and content of a MEA can draw on a wide range of disciplines or subjects. Collaboration in teams is essential to benefit from multiple perspectives and drive the express, test, and revise cycle. Both PBL and MEA pedagogies enable students to become more proficient at self assessment. The classical format for implementing a problem in a PBL environment is to present the problem before the students have learned the knowledge to solve the problem. This allows students to discover the need for the knowledge. MEAs, on the other hand, are presented with the intention that students (re)construct the knowledge for themselves. So the timing of the presentation of the problem could be before or after they have seen the knowledge domain needed to solve the problem.

MEAs also show some similarities to Learning by Design [8]. Both pedagogies present problems with constraints, multiple competing variables, and measurable criteria for success.

The solution (design or process) is interactive through artifacts; in the case of MEAs, the artifact is the mathematical model. Both require iteration - expression, testing, and revision of the product, reflection, and collaboration. Both can depend on representation as tools for thought and problem solving – drawings, sketches, and models – with the level of abstraction depending on the problem. LBD and MEA pedagogy do not rely on inquiry, rather the process of solving the problem provides feedback to improve the solution.

Educational Versatility and Versatile Implementation

As we have developed and implemented MEAs, we have found that they can be used in a variety of educational settings to address multiple educational, institutional, and national objectives for student learning and engagement at a number of academic levels in a variety of academic programs. To demonstrate the versatility of MEAs, we discuss the whys and hows of implementation in a first-year engineering course and in secondary school classrooms.

First-Year Engineering

First-year engineering programs are charged with introducing students to the engineering disciplines and practice of engineering, placing students on a trajectory for attaining skills and understandings required for graduation in engineering, and retaining students with potential for success in engineering. We have found that carefully constructed MEAs implemented in the first-year can facilitate fulfillment of this charge.

Implementation

We instruct 1400 students each fall and 300 students each spring in a required 2-credit hour first-year engineering problem solving and computer tools course. Lecture divisions of up to 450 students are led by faculty and meet twice each week for 50 minutes. Labs of 28-32 students are led by one graduate teaching assistant with the support of an undergraduate teaching assistant and meet once each week for 110 minutes. The course is designed to give students an appreciation for what the pursuit of an engineering degree entails. The course learning objectives are such that students successfully completing the course are able to:

- Develop a logical problem solving process which includes sequential structures, conditional structures, and repetition structures for fundamental engineering problems,
- Translate a written problem statement into a mathematical model,

- Solve fundamental engineering problems using computer tools,
- Perform basic file management tasks using an appropriate computer tool,
- Work effectively and ethically as a member of a technical team, and
- Develop a work ethic appropriate for the engineering profession.

The syllabus is a coordinated mix of introduction to engineering fundamentals, including graphical representation, statistics, and economics, and introduction to computer tools used to solve engineering problems, specifically MATLAB® (a computational tool and interpreted programming language used by engineers), Excel®, and UNIX.

The size of the course, limited physical space, and the low number of course-credit hours - and therefore contact hours, presents challenges to implementing open-ended problems. Our aim was to develop MEAs that could be used in the lab setting and that are perceived as representative of what engineers do in practice, are tightly aligned to the course content and learning objectives, are mathematically significant, and support students' development of abilities to work on a technical team.

Four MEAs have been implemented each semester since Fall 2002 [9]. MEAs are conducted in the lab facilitated by the teaching assistants (TAs). Student teams of four work through the MEA in 45-60 minutes to produce a written response to the client, often in the form of a memo, that explains their mathematical model. Often MEAs are used to launch a model-development sequence in which student teams continue to work with their model through a *model-exploration activity* and a *model adaptation activity* over the course of the next week (as homework) or 4-6 weeks (as a project) [2]. In short, the model-exploration activity introduces the students to a method engineers use to solve the problem and asks them to compare their method to the engineering method. The model-adaptation activity consists of having the students create a computer tool to implement their procedure or the engineering method. The ACS MEA launched a modeling sequence that addresses all course learning objectives [2].

We have found that training of both the teaching assistants and the students is necessary for successful implementation. MEAs, teaching students in teams, and the role of the facilitator are not familiar to most of the teaching assistants. Their buy-in to the pedagogy is critical since they are the sole instructors in the lab. We have developed a training program aimed

at engaging the TAs in the MEA development and implementation process. The week before classes begin, TAs work through the first MEA they will use in the lab, learn about principles for MEA design and how the MEAs link to the course learning objectives, and discuss methods for interacting with student teams during an MEA. Over the course of the semester, we maintain an open dialogue with the TAs. Before an MEA is used in their classrooms, the TAs work through the MEA in teams and provide feedback to the development team on the text, content, and underlying engineering concepts of the MEA and potential assessment strategies.

Most students have little experience with open-ended problems much less with working on technical teams in a student-centered learning activity. The faculty need to work with the students to set appropriate expectations for their individual and team performance and engagement. In lecture, before the students see an MEA in lab, the faculty cover basic teaming concepts and conduct an MEA (with 450 students!). Then the faculty walk the students through the purpose of MEAs, the role of the TAs during MEAs, and expectations for all involved. We have found that this alleviates some student frustration with an unfamiliar learning situation. By the second MEA conducted in the lab, students have a decent understanding of the logistics and expectations.

Introduction to Engineering

First-year engineering students tend to be ill-informed or uninformed about many engineering disciplines. Consider that the reality principle places the problem in an engineering context. This context can be used as a vehicle for introducing various engineering disciplines as well as emerging technologies and fields (e.g., nanotechnology). In the case of the ACS MEA, we provided a problem solving experience typical for material science engineers. First-year students tend to know very little about this discipline. This MEA context provides the opportunity for the students to learn first-hand some engineering concepts used in this major. First-year students also tend to have a very narrow view of the nature of engineering practice. They know that math, science, and problem-solving will be involved, but how is unknown. In addition, many students have never solved truly open-ended problems, and they are uncomfortable with the process of generating a solution when there isn't one "right" answer. MEAs can begin to reform students' notions of modeling and problem-solving. The MEA construct documentation principle ensures that students build

a mathematical model. Most first-year students do not consider a procedure, an explanation, or representation a model. MEAs provide a platform for discussing and engaging students in how engineers view, create, and use models. The self-assessment principle ensures that the problem has cues built-in that guide the student to the fulfillment of the client's needs and development of a workable solution.

Further, the need for professional skills, for instance communication skills, is not in students' purview. MEAs provide a means for students to engage in a greater range of activities that can help them better understand the work of engineers. The construct documentation principle requires students to document their solution, highlighting the need for engineers to keep records of their design work and effectively communicate to their client. MEAs are designed to be solved by technical teams, encouraging students to develop better verbal communication skills and effective team maintenance habits within a context similar to the engineering workplace where they will eventually need those skills. So, the professional skills are intertwined with the technical engineering content.

Attaining Skills and Understandings for Graduation in Engineering

The nature of MEAs offers opportunities to begin to address the ABET Criterion 3 a through k [10] and prepare students for their upper-division coursework. Table 1 details how each criteria is or can be addressed. It is acknowledged that the degree to which a single MEA can address each criterion varies according to the content, context, and implementation of the problem. We often use an MEA to launch a model-development sequence [2] that allows us to better address both course learning objectives and ABET criteria. Due to the nature and goals of our first-year engineering problem solving course, the MEAs and model-development sequences we have developed and implemented emphasize (a) the application of mathematics, (b) analysis and interpretation of data, (d) teaming, (e) engineering problem solving, and (k) application of computer tools. The contexts we have chosen have allowed us to touch on (f) the engineering profession and ethics, (g) communication, (h) global and societal context, and (j) contemporary issues.

Retaining Students

Retention of a diverse student population in engineering is a high priority. MEAs allow for the creation of learning environments

Table 1. Addressing ABET Criteria with MEAs

ABET Criterion 3 a-k [10]	How it is addressed by an MEA
(a) an ability to apply knowledge of mathematics, science, and engineering	In first-year engineering problems, students draw on their knowledge of algebra, trigonometry, geometry, dimensions, and units. Statistics might also be applied. Students apply more advanced knowledge as the contexts become increasingly complex.
(b) an ability to design and conduct experiments, as well as to analyze and interpret data	While MEAs are not experimental in nature, students are often called on to develop a model for interpreting data and making decisions.
(c) an ability to design a system, component, or process to meet desired needs	While these are not classical engineering design problems, students are in essence designing a procedure, explanation, or representation. In so doing, students go through a series of express, test and revise cycles, which are fundamental to the design process. The model-development sequence [2] can be formulated to allow the engineering design process to be highlighted.
(d) an ability to function on multi-disciplinary teams	The development of team interaction skills is fostered as student teams create their solution and work through the MEA and model-development sequence [2]. In a first-year course, we often have a high degree of variability in math, computer, and professional skills. Team composition can be manipulated to encourage students to draw on each other's strengths. But care must be taken to encourage student learning and participation in all facets of the activity.
(e) an ability to identify, formulate, and solve engineering problems	Students need to interpret the client's needs to better define and solve the problem. In fact, understanding the client's purpose and need serves as a mechanism for self-assessment as they progress on their work. MEAs can provide the instructor with an opportunity to highlight this step in the engineering problem solving method.
(f) an understanding of professional and ethical responsibility	The MEA's have been embedded into a classroom culture where professionalism has been explicitly taught. For example, the course explicitly addresses the notion of teaming and communication, and the accompanying responsibilities. Further, many of the MEA's have been designed to include contexts and background information that address professional and ethical responsibility. While not emphasized in the problems we have constructed to date, one could envision a debriefing focused on the impact of creating a faulty or unreliable procedure.
(g) an ability to communicate effectively	Students practice their written communication skills through their written responses to the client. They also learn to communicate their technical ideas by working interactively with each other – rehearsing the explanations that may eventually be included in their documentation.
(h) the broad education necessary to understand the impact of engineering solutions in a global and societal context	In some of our work, we have deliberately designed the activities to include contexts that illuminate global and societal considerations.
(i) a recognition of the need for, and an ability to engage in life-long learning	Because students have to learn as they design the procedure for the local context of the problem, it provides an opportunity for them to experience the need for dynamic learning as they work in situ.
(j) a knowledge of contemporary issues	By design, we select topics that incorporate contemporary issues, such as nanotechnology.
(k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice	Model-development sequences [2] should drive the need for the use of computer tools to implement the solution to complex problems. We've used MEAs as a motivator for using computer tools to adapt their models. MEAs also provide opportunities for students to draw upon techniques, skills and concepts they have learned and to modify that knowledge to adapt to the new situation.

that encourage and support the success of all students. By incorporating class work that is more aligned with what is needed for success in engineering, students who are talented, but do not fit conventional classroom profiles of “excellence”, have opportunities to show what they know and can do [11]. The incorporation of modeling activities into the classroom has the potential to help instructors recognize diverse and creative thinking in students and provides all students with early access to powerful ideas and tools used in engineering. These problems demonstrate the relevance of math, science, and technology by presenting problems in contexts that are relevant and related to students’ interests or the real world. These problems also demonstrate that the combination of technical and professional skills is valued.

Lab time in the first-year problem solving course is divided between MEAs and more conventional, scaffolded, hands-on problem-solving experiences using computer tools. In our preliminary data analysis of students’ reactions to the labs, we have learned that while all students are appreciative of the lab experiences, women and international students reactions were especially positive [12]. Further, women were especially positive toward group work, although they still report less positive feelings of “worth” and “efficacy” with respect to their group compared to men. The initial findings indicate that the lab experiences are worthwhile endeavors to tap the strengths and interests of all students, especially women and internationals.

Secondary School Mathematics

The National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* [13] emphasizes problem solving, connections, reasoning and communication as overarching goals for students’ experiences in mathematics classes. Thus, in addition to students’ creation of mathematical models for a local problem situation, teachers are also able to address the overarching NCTM goals that are often not readily addressed in conventional textbooks. Further, by adapting engineering-based MEAs for use in middle and high school, students (and their teachers) gain an opportunity to learn about engineering as a potential career.

In the secondary mathematics classroom, the primary goal of most teachers is to address specific mathematics standards set at the local or state level, which usually align well with the NCTM *Standards*. Usually the approach to accomplishing these standards is to develop a

checklist of concepts, skills and processes, and to teach them one-by-one. However, middle and high school teachers who have been exposed to model-eliciting activities in summer institutes and professional development sessions recognize that although these activities take quite a bit of time (usually about 3 days), students will engage in multiple mathematics standards during that time. The ACS MEA, in particular, requires students to create a mathematical model (a procedure for determining relative crystal size). During the model creation process, students use proportional reasoning and apply measurement concepts and skills. Other process standards that are addressed in substantive ways are standards associated with mathematical communication, problem solving and connections. Mathematical communication takes place in substantive ways as students collaborate with each other as they pose trial solutions, test them, and then revise them. Further, the final product (a memo to the client) requires them to formally communicate the mathematical procedure they have devised. The benefit is that the communication standard is addressed naturally in purposeful situations. The problem solving standard is clearly addressed, since students must bring their ideas to bear in a new and unfamiliar situation; students are not simply identifying a pre-learned procedure to apply but creating a set of steps that will result in a sensible procedure that can be used by other people and on other similar samples. The “connections” standard encourages connections to be made within mathematical topics as well as to the real world and other disciplines. Within mathematics, the students must integrate and simultaneously use (and create) ideas about proportions, measurement, and mathematical modeling as they express, test and revise trial solutions, and the final integration of these aspects of mathematics are revealed in their final product. The ACS MEA is also an integrated activity – in particular a blending of mathematics and science, since the mathematical model students are creating is one that would be used to determine scientific properties of the material.

While the ACS MEA has been used in a variety of middle and high school setting, a noteworthy example was the use of this activity by 15 inner city junior and senior high school students in an after school program in 2003. These students were voluntarily participating in a six week program in which they worked after school for 1.5 hours on Wednesdays and Thursdays. Each week was devoted to one MEA. The ACS MEA was the fourth activity of the program.

The purpose of the after-school program was to introduce teachers to the use of MEAs in an environment where standardized testing pressures were non-existent, and where students were likely to be engaged in the activities. All of the students were of African American ethnicity; gender was split 7 girls and 8 boys. The three teachers (2 African American, 1 White Immigrant) were engaged in weekly professional development sessions on Tuesdays, in which they would solve the problem and make plans to implement the problem in the subsequent two days. As a result of this experience, all of the students volunteered to talk with Purdue faculty about their experience the following autumn, reported vivid memories about the activities, and explicitly talked about the meaningfulness of the experience. The teachers' enthusiasm has since been caught by the rest of the high school teachers, and model-eliciting activities are being systemically worked into the regular curriculum of the school: four MEAs at each of the four levels of mathematics classes (freshman, sophomores, juniors and seniors).

Challenges and Rewards to Model-Eliciting Activity Design

The first challenge in model-eliciting activity design is selecting an appropriate context and then narrowing the problems within that context so that they are accessible to first-year engineering students. For example, within the ACS MEA, we only presented a brief introduction to stereology and the materials science concepts related to the context. However, students should have experience measuring size in other contexts as well as knowledge of basic statistical procedures to analyze the data. However, we maintain a delicate balance between keeping the context simple enough that students can self-assess the models they construct and keeping enough of the context to make the problem realistic and avoid neglecting important engineering education concerns. One means of resolving this challenge is the collaboration between engineering and education faculty and graduate students. The engineers bring knowledge of realistic engineering problems and the educators bring understanding of students and student learning to aid in the scaling of those contexts into a feasible MEA. However, the rewards to using realistic engineering contexts are that we can present students with complex problem solving activities that actually require a team to solve. So, students are placed in a problem solving situation which encourages them

to draw on the experiences and knowledge of their teammates. We are also helping students to learn what engineers do and how they work.

The next challenge in the design is crafting a task that the students can work on as autonomously as possible in order to fulfill the self-assessment principle. This means we carefully consider the language and construction of the problem statement and background information. Particularly since first-year students have little knowledge of engineering, we have to choose the technical language carefully and define terms when necessary. Language is also important when considering large populations of international students, students from rural and urban environments, gender and ethnic differences, and the diverse set of experiences students may have had in high school. We also have to be very clear about the product we are asking them to create so they can evaluate their responses and methods. In realistic problem-solving situations, there is often not enough or too much data so we also evaluate the amount and types of data necessary to make the task realistic but solvable. The reward of this process is that very little should have to be done to the problem statement after the students have received it. In a course with 1400 students taught by 18 different TAs, this means we can maintain as much uniformity between laboratory sections as possible. In addition, the TA should be able to refer most questions about the task back to the information in the materials given to the students.

The final challenge in designing the tasks is to maintain a balance between open-endedness and having every possible answer as a "right" answer. So, we avoid tasks where the procedure is simply stringing together the correct set of mathematical equations resulting in one right answer. We would rather have students construct their own equations as mathematical models. We also have to select tasks that will encourage self-assessment so that the students' first response will not be their final response. By encouraging an iterative problem solving process, we anticipate that students will brainstorm initial ideas and then we want them to assess those ideas. For any task we construct, some solutions should be better than others and the students should be able to evaluate how and why some solutions are better than others. The correctness of any procedure is evaluated by its match to the client's needs. The reward to this is that the students have to go through an authentic engineering design process where the relative advantages, disadvan-

tages, limitations, and strengths of procedures are considered within a context. The students also learn that most realistic problems do not have one right answer and that correctness is often determined by the context and the needs of the client.

Conclusions

Model-eliciting activities provide educators with an opportunity to address complex educational objectives at a number of academic levels. The principles of MEA design enable us to create meaningful, open-ended, engineering-based problems that are accessible to all students and prompt a problem-solving process employed by engineers. MEAs drive a need for the development of professional skills, like teaming and communication, in the classroom in a non-artificial way. They provide relevance for often decontextualized science and technology topics. By providing our students with rich problem solving experiences, we hope to engage all students and retain those with potential for success in STEM fields. Most of all, the implementation of MEAs encourages us to adopt a new way of thinking about teaching and student learning.

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Appendix: Part I: Model-Eliciting Activity (as Distributed to Students)

Batter, Batter, . . . SWING!!

Osceola, IN – The Lady Panthers are ready to pounce! Coach Greg Meyers verified today that he will be forming a new summer league softball team, the Lady Panthers, for girls 12 to 13 years old.

“We have been signing up players, and we still have two positions open – third base and centerfield. So, if you know of anyone that might be interested in playing these positions or even other positions, please have them contact me,” said Meyers. “We are also beginning to make decisions about our uniforms and the pieces of equipment that we need to purchase.”

The Lady Panthers will wear uniforms of yellow and black after their team colors. Harry’s Sport Shop on Main Street is designing the uniforms, and the uniforms will be available for purchase by next Friday. Players will be responsible for purchasing their own uniforms, cleats, and mitts. Harry’s will also have available other Lady Panthers items such as baseball hats, keychains, and T-shirts for Lady Panther fans.

Since deciding on the team’s colors and the uniforms, Coach Meyer has been investigating the purchase of the necessary equipment for practice and games. He has already purchased plenty of softballs for the team and has been pricing batting helmets. Gart Brothers Sports has helmets available for \$34.99 and Outpost Sports has them available for \$32.95.

“I’ll probably purchase the helmets from Gart Brothers because they are of better quality than the helmets available at Outpost,” said Coach Meyers. “Besides, I can pick up the helmets when I also purchase the catcher’s mitt and the catcher’s mask from Gart’s.”

The only remaining equipment for the coach to purchase will be the softball bats. Currently, he has found three styles of aluminum bats that he likes and that cost the same amount. All three styles are available at Harry’s Sport Shop.

“Since bats are so expensive and last year the bats dented too easily, I want to purchase bats that are more resistant to denting,” commented Coach Meyers.

The first game for the Lady Panthers will occur on June 6 at home. They will be playing the Nappanee Ravens at Strawberry Field.

“I’m looking forward to helping the girls get ready for our first game. I’ve heard the Nap-

panee Ravens have some good players, so we’ll need to be ready to go!” explained Coach Meyers.

We want to wish good luck to Coach Meyers and the Lady Panthers in their game against the Ravens and in their upcoming season!! Take ‘em out with a growl, ladies!

Background Information: Part A

All metal is composed of crystals, and it is possible to see metal crystals without a microscope. Check out the metal poles supporting a traffic light on a nearby West Lafayette corner (Figure A.1). These steel poles are coated with a thin layer of zinc metal that helps prevent rust formation. The zinc metal forms very large crystals that can be readily seen by the eye. The pictures in Figure A.1 and A.2 show the metal pole and a close-up picture of the crystals on the surface of the pole, respectively. The letters a, b, and c indicate three crystals that have had a line drawn along the boundaries between the crystals. The arrow on each picture is the scale marker.

Background Information: Part B

The size of the crystals in aluminum is often a good indicator of the relative resistance to denting (or strength) of the material. Aluminum consisting of small crystals is stronger than aluminum consisting of large crystals. Figure A.3 shows microscopic photographs of crystals for three samples of aluminum metal. They are called ‘micrographs’ because they provide a standard way to compare the size of crystals in aluminum. Materials engineers can chemically treat polished pieces of aluminum to make the boundaries between the crystals more visible. Using a camera attached to a microscope, a picture of the boundaries between the crystals can be obtained, and then the size of the crystals can be estimated.

Model-Eliciting Activity

Imagine that the engineering firm you work for is competing for a job for a company that produces aluminum and sells it to manufacturers, such as makers of softball bats. Your potential client (the aluminum producer) wants to improve quality control over their aluminum manufacturing process. To win the job, your engineering firm needs to propose an algorithm

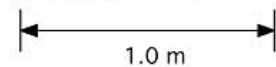


Figure A.1. Traffic light pole

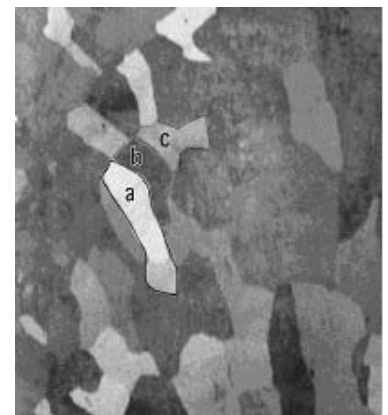


Figure A.2. Close-up of crystals

for determining the size of aluminum crystals from micrographs.

Your supervisor wants your technical team to develop and articulate a procedure that will determine the average size of crystals for samples of aluminum.

Using the WebCT discussion board designated for your team, develop a procedure that will win the job. Your team's final solution to this problem should include:

- a. A series of steps that can be used to determine an indicator of average crystal size for samples of aluminum using micrographs.
- b. A description of how the procedure would work by applying it to samples A, B, and C shown in Figure A.3.

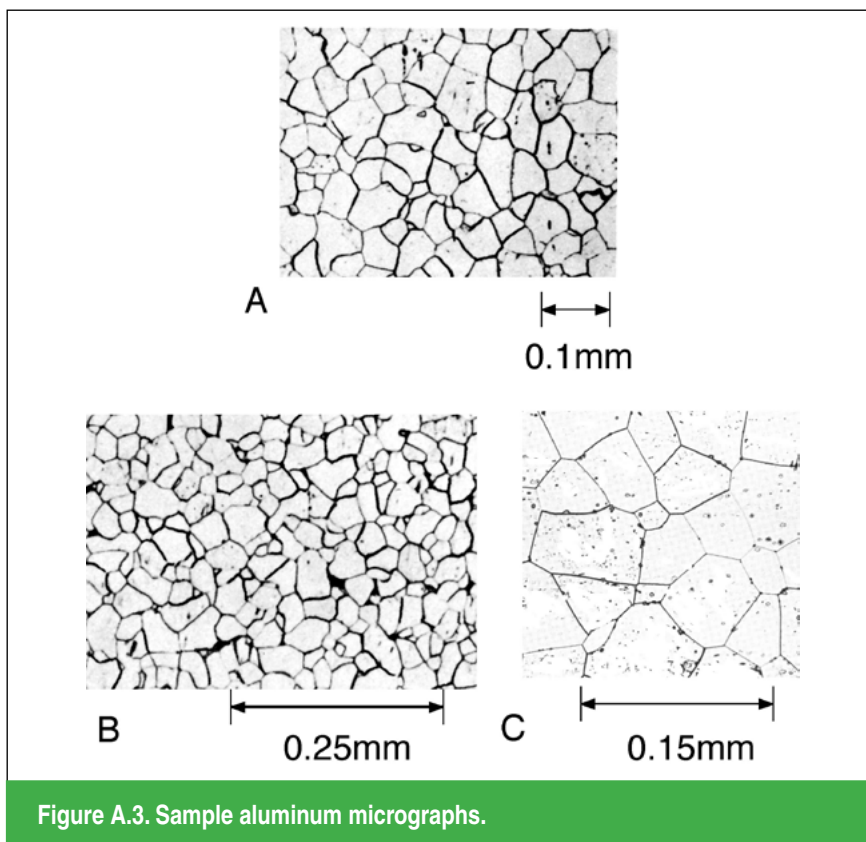


Figure A.3. Sample aluminum micrographs.