Interdisciplinary Lively Application Projects in Calculus Courses

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ABSTRACT

We report on a two-year NSF-funded project to strengthen connections among science, technology, engineering, and mathematics (STEM) disciplines. One component of this project was to produce some initial data on the effectiveness of Interdisciplinary Lively Applications Projects (ILAPs) in teaching science and engineering undergraduates. ILAPs are interdisciplinary group problem-solving projects, co-written by mathematics faculty and science/technology/engineering faculty. These small group projects are designed to foster student interest by being lively, real-world applications of mathematics in science and engineering. ILAPs are intended to assist students in learning to communicate across disciplines and in developing problem-solving skills. We summarize our development and use of ILAPs in the calculus courses and, in particular, describe our assessment data and results on the effectiveness of ILAPs in learning and related results.

I. INTRODUCTION

This project, "Enhancing Interdisciplinary Interactions in the College of Engineering and Natural Sciences", at The University of Tulsa (TU) had several goals. The main objectives were to develop and implement interdisciplinary lively application projects (ILAPs) [1] in order to assist STEM students in learning and STEM faculty in teaching [2], and to produce some initial assessment data on the effectiveness of ILAPs in learning.

The concept of ILAPs originated from a consortium of twelve schools led by the United States Military Academy with an NSF funded project, Project INTERMATH [3]. ILAPs are interdisciplinary group problem-solving projects designed for undergraduates, co-written by mathematics faculty and science/technology/engineering faculty. These small group projects were designed to foster student interest by being lively. "Being lively" means that students are actively involved in and outside of class with project problem-solving and/or hands-on activities. ILAPs can motivate students to understand the connections between mathematical tools/concepts and applications within the broader science and engineering fields. With these projects STEM students can see real-world applications of mathematics in science and engineering. A description of the ways that ILAPs have been integrated into some of the Project INTERMATH institutions can be found in [3].

One of the main goals of the project was to produce assessment data on the effectiveness of ILAPs in learning. There is much anecdotal evidence to support claims that students benefit in many ways from ILAPs. For example, ILAPs demonstrate how mathematics is used in partner disciplines [4], give students experience in working and communicating as part of an interdisciplinary team, provide practical experience in the use of technology, etc. However, formal assessments on the pedagogical effectiveness of ILAPs and similar projects are only just beginning to appear in the literature [5]. This project makes an initial contribution to such analyses.

II. PROJECT DESIGN, ACTIVITIES, AND ASSESSMENT

The project was designed to introduce ILAPs into the mathematics curriculum by phasing in their implementation over several semesters. In the first academic year, ILAPs from various engineering and science disciplines were introduced into Calculus I and Calculus II. There were two ILAPs in the fall semester of Calculus I and two ILAPs in the spring semester of Calculus II. The spring sections of Calculus I were not part of this project because the students in those sections tended to be mostly biology majors, whereas the targeted students were predominantly engineering, natural science, and computer science majors. In each course, one section of the course was assigned a traditional calculus project instead of the ILAP in order to

Question and SCALE: 1 Strongly Disagree 6 Strongly Agree	Time on Project	# of Group Meetings	Class Intro Helped Understanding	Satisfied With How Group Worked Together	Helped Make Connection Between Calculus and Real World	Gained a Better Understanding of How Derivatives are Used	Project Was Interesting	Would Choose Hands-On Project Over Math Based
ILAP Median	7 hours	4 meetings	5	5	4	4	4	5
Non-ILAP Median	8 hours	4 meetings	4	5	4	5	4	5

Table 1. FALL 2005 CALCULUS I MATHEMATICS-ELECTRICAL ENGINEERING ILAP VERSUS NON-ILAP PROJECT 1 RESULTS:

Question and SCALE: 1 Strongly Disagree 6 Strongly Agree	Time on Project	# of Group Meetings	Class Intro Helped Understanding	Satisfied With How Group Worked Together	Helped Make Connection Between Calculus and Real World	Gained a Better Understanding of How Derivatives and Integrals Are Used in Working with Position, Velocity, and Acceleration	Project Was Interesting	Would Choose Hands-On Project Over Math Based
ILAP Median	4 hours	3 meetings	6	5	4	5	5	5
Non-ILAP Median	9 hours	4 meetings	4.5	5	4	4	4	5

Table 2. FALL 2005 CALCULUS I MATHEMATICS-PHYSICS ILAP VERSUS NON-ILAP PROJECT 2 RESULTS:

provide a control group for evaluation and assessment. The traditional projects were similar to the projects in the course text [6] that were not interdisciplinary, applied, or hands-on. Using the results and recommendations from the first year, ILAPs were introduced into Calculus III in the second year, and the ILAPs for Calculus I and Calculus II were refined and implemented (see Section VII). In all, six new ILAPs were created and two existing ILAPs were adapted (from [3]).

The assessment goals were to assess the effectiveness of ILAPs, determine what factors impact the effectiveness, and make recommendations for future implementation in the mathematics and engineering curricula. Assessment was provided and coordinated by an external independent evaluator, Douglas Grouws, Mathematics Education, University of Missouri, who worked with a TU graduate student, Leslie Keiser. These project evaluators collected and analyzed assessment data on the effectiveness of ILAPs in learning. ILAP/non-ILAP questionnaires were administered, one-on-one student interviews on student attitudes with respect to the ILAPs versus non-ILAPs (control group) were carried out, and project-based questions were included on tests. See the 2005 paper [7] for student comments. Student comments from the one-on-one interviews were similar to student comments from the questionnaires.

Additionally, a Concepts of Mathematics Inventory (CMI) was administered as a pre-test to calculus students (at the beginning of Calculus I, fall 2004) and administered later as a post-test (at the end of Calculus III, fall 2005). This questionnaire was used to test for changes in student attitudes toward mathematics, which resulted due to general exposure to the calculus courses.

III. ILAP/NON-ILAP SCHEDULE AND PROJECTS

Students were given information packets containing the following items: project description and assignment, grading policy and related information, technical report format and writing guide, sample report, information on working effectively in small groups, first group meeting form, and small group peer evaluation forms. This additional information, ILAPs, and supporting items are available for download at the project web site, http://www.ilaps.utulsa.edu/.

A typical class size was approximately 40 students. Teams consisting of two to four students worked on each project for two weeks (team size varied depending on the specific ILAP/non-ILAP). The teams were randomly chosen. Students were introduced to the projects in classes, but did most of their work outside of classes. There were usually two weeks between the completion of the first project and assignment of the second project in those courses in which two projects were assigned. There were also control classes of students who did projects that were not interdisciplinary and not hands-on. During this two-year project we team-taught nine Interdisciplinary Lively Applications Projects:

Fall 2004 – Calculus I ILAPs

Electrical Engineering-Mathematics ILAP: Designing an electric car - RC circuits and exponential growth/decay.

Chemistry-Mathematics ILAP: Chemical kinetics - decay of phenolphthalein in the presence of sodium hydroxide and exponential decay.

Spring 2005 – Calculus II ILAPs

Physics-Mathematics ILAP: Planck's law for blackbody radiation - A Mathematica project.

Mechanical Engineering-Mathematics ILAP: Beam deflection using real-time sensors.

Fall 2005 – Calculus I ILAPs

Electrical Engineering-Mathematics ILAP: An electric car - RC circuits and exponential growth/decay revisited.

Physics-Mathematics ILAP: Introduction to the special theory of relativity using Mathematica: Galilean versus Lorentz transformations.

Fall 2005 – Calculus II ILAP

Chemistry-Mathematics ILAP: Saving a drug-poisoning victim [8] - exponential decay and related differential equations.

Fall 2005 – Calculus III ILAP

Geosciences-Mathematics ILAP: Strain tensor, displacement vector, and deformation matrices; vector and tensor calculus.

Spring 2006 – Calculus II ILAP

Chemical Engineering-Mathematics ILAP: Wastewater treatment facilities - Curve fitting and integration [3].

IV. RESULTS FROM ILAP VERSUS NON-ILAP STUDENT REACTIONS FALL SEMESTER 2005

Students completed questionnaires after submitting each ILAP/non-ILAP, but before receiving their grades. The student questionnaires were developed by Douglas Grouws and Leslie Keiser (see Section II). The questions and assessment topics from the questionnaires used for Tables 1–5 follow.

Questions and assessment topics for Tables 1, 2, 4, and 5:

1. How much time (in hours) did you spend working on this project?
2. How many group meetings did you have?
3. The class introduction helped me understand the project better than if I had just read through the project description.
4. I was satisfied with how my group worked together on this project.
5. This project helped me make a connection between calculus and the real world.
6. By completing this project, I gained a better understanding of *<topic(s) of project>*.
7. This project was interesting.
8. I would like to do more of these kinds of projects in the mathematics courses I take.

The questions used to gather the data for Table 3 differed from the questions used for Tables 1, 2, 4, and 5 because the questionnaire for Table 3 was given to students at the end of Calculus I, after students had experienced two projects (not just one), either two ILAPs or two non-ILAPs. Questions and assessment topics for Table 3:

1. The projects this semester gave me a better understanding of how the applications of calculus relate to other science and engineering disciplines.
2. In the future, including more hands-on aspects to the projects would be beneficial.
3. In the future, interdisciplinary science and

Question and SCALE: 1 Strongly Disagree 6 Strongly Agree	Projects Gave Better Understanding of How Applications of Calculus Relate to Other Disciplines	In the Future Including More Hands-on Aspects to the Project Would Be Beneficial	In the Future Interdisciplinary Aspects to the Projects Would Be Beneficial	Group Work Nature of Projects Was a Valuable Experience	Projects Gave Confidence in Ability to Retain the Mathematics Involved in Each Project	Did Projects Affect Attitude Toward Calculus in a Positive or Negative Way 1= Neg 6=Pos
ILAP Median	5	5	5	4.5	4	3
Non-ILAP Median	5	5	4	4	4	3

Table 3. FALL 2005 CALCULUS I ILAP VERSUS NON-ILAP FINAL RESULTS (AFTER TWO PROJECTS):

Question and SCALE: 1 Strongly Disagree 6 Strongly Agree	Time on Project	# of Meetings	Class Intro Helped Understanding	Satisfied with How Group Worked Together	Helped Make Connection Between Calculus And Real World	Gained a Better Understanding of Exponential Growth and Decay and Differential Equations	Project Was Interesting	Would Choose Hands-On Project Over Math Based
ILAP Median	10 hours	4 meetings	4.5	5	5	5	4	5
Non-ILAP Median	8.5 hours	4 meetings	4	5	4	5	4	5

Table 4. FALL 2005 CALCULUS II MATHEMATICS-CHEMSTRY ILAP VERSUS NON-ILAP PROJECT RESULTS:

engineering aspect to the projects would be beneficial.

4. The group work nature of the projects this semester was a valuable experience.
5. The projects this semester gave me more confidence in my ability to retain the mathematics involved in each project.
6. Did the projects this semester affect your attitude towards calculus in a positive or negative way? (Negative=1, Neutral=3, Positive=5, etc.)

Numerical responses to assessment topics were assigned the following interpretations: 1-Strongly disagree; 2-Disagree; 3-Slightly disagree; 4-Slightly agree; 5-Agree; 6-Strongly agree. (See Tables 1, 2, 4, and 5, values in columns 4–9; see Table 3, values in columns 2–7.)

Results from administering questionnaires on the 2004 ILAPs and associated non-ILAPs can be found in the 2005 paper [7]. Results from administering the questionnaires on the 2005 ILAPs and non-ILAPs are given in Tables 1–5.

From Tables 1–5, it can be observed that the student preference results for the ILAPs are generally comparable to those for the non-ILAPs (headings highlighted in green indicate different median response values to questions for the ILAP student cohort versus non-ILAP student cohort). In each table, there are between one and three categories (other than time spent on project and number of group meetings) for which median response values from the ILAP cohort differ from those from the non-ILAP cohort, and in the cases for which the values differ, they differ at most by only one or two points. Our analysis of the differences in results, organized by course, is given below.

Fall 2005 Calculus I:

Table 1. Fall 2005 Calculus I; Math-Electrical Engineering ILAP versus Non-ILAP Project 1:

The fall 2005 Calculus I questionnaire results from the Mathematics-Electrical Engineering ILAP students versus the corresponding results from the non-ILAP students show differences between these two groups with respect to three responses, time spent on the project, whether

Question and SCALE: 1 Strongly Disagree 6 Strongly Agree	Time on Project	# of Group Meetings	Class Intro Helped Understanding	Satisfied With How Group Worked Together	Helped Make Connection Between Calculus and Real World	Gained a Better Understanding of How Matrix Transformations Can be Used to Describe Changes in the Geometry of an Object	Project Was Interesting	Would Choose Hands-On Project Over Math Based
ILAP Median	4 hours	3 meetings	5	6	4	5	4	5
Non-ILAP Median	4.5 hours	2 meetings	6	6	2	5	4	4

Table 5. FALL 2005 CALCULUS III MATHEMATICS-GEOSCIENCES ILAP VERSUS NON-ILAP PROJECT

the class introduction helped in understanding the project, and whether a better understanding of the use of derivatives was gained from the project.

Over a two week period, the ILAP cohort spent slightly less time on their project, 7 hours, than the non-ILAP cohort, which spent 8 hours. The median response values indicate that the ILAP cohort agreed that the class introduction helped them understand the material, versus the non-ILAP cohort, which only slightly agreed. This may reflect the fact that the ILAP had an explicit electrical engineering application, while the non-ILAP did not. Perhaps the ILAP class introduction was necessary in order for students to interpret the electrical engineering application. The ILAP cohort responded that the project only slightly helped them gain a better understanding of the project topic (examples of how derivatives are used), versus the non-ILAP cohort, which agreed that they had gained a better understanding. This result was unexpected. The project faculty had expected that the ILAP, with its applied and hands-on aspects (running the electric "cars" and comparing predicted theoretical results with experimentally obtained results) would reinforce student understanding. However, it may have been that instead of viewing the theory and application as two facets of the same project, the students got "distracted" by the hands-on component of the project.

Table 2. Fall 2005 Calculus I; Math-Physics ILAP versus Non-ILAP Project 2:

The fall 2005 Calculus I questionnaire results from the Mathematics-Physics ILAP students versus the corresponding results from the non-ILAP students show differences between these two groups with respect to five responses, time spent on the project, number of group meetings, whether the class introduction helped in understanding the project, whether a better understanding of the use of derivatives and integrals in kinematics was gained from the project, and whether the project was interesting.

Over a two week period, the ILAP cohort spent much less time on their project, 4 hours (with 3 meetings), than the non-ILAP cohort, which spent 9 hours (with 4 meetings). The median response values indicate that the ILAP cohort strongly agreed that the class introduction helped them understand the material, versus the non-ILAP cohort, which only slightly agreed/agreed. The ILAP cohort agreed that the project helped them gain a better understanding of the project topic (examples of how calculus is used in kinematics and/or relativity) and that their project was interesting, versus the non-ILAP cohort, which only slightly agreed that they gained a better understanding and that the project was interesting. The ILAP investigated relativity, which some students had previously studied in their physics courses, and the physics instructor gave a very lively presentation which involved student participants. These factors may account for the more positive responses from the ILAP cohort.

Table 3. Fall 2005 Calculus I; ILAP versus Non-ILAP Final Results (after Projects 1 and 2):

The fall 2005 Calculus I questionnaire results obtained from students at the end of the course, after both pairs of projects, ILAP/non-ILAP #1 and ILAP/non-ILAP #2, show differences between the ILAP versus non-ILAP groups with respect to two responses, whether future interdisciplinary aspects to projects would be beneficial and whether the group work was a valuable experience. The ILAP cohort median response values were higher in both these categories. The ILAP cohort agreed, while the non-ILAP cohort only slightly agreed, that interdisciplinary aspects would be beneficial. The ILAP cohort slightly agreed/agreed, while the non-ILAP cohort only slightly agreed, that the group nature of the projects was valuable. These types of results lend support for the use of ILAPs in Calculus II to enhance student learning.

Fall 2005 Calculus II:

Table 4. Fall 2005 Calculus II; Math-Chemistry ILAP versus Non-ILAP Project:

The fall 2005 Calculus II questionnaire results from the Mathematics-Chemistry ILAP students versus the corresponding results from the non-ILAP students show differences between these two groups with respect to three responses, time spent on the project, whether the class introduction helped in understanding the project, and whether a better understanding of some connections between calculus and the "real world" was gained from the project.

Over a two week period, the ILAP cohort spent slightly more time on their project, 10 hours, than the non-ILAP cohort, which spent 8.5 hours. The median response values indicate that the ILAP cohort slightly agreed/agreed that the class introduction helped them understand the material, versus the non-ILAP cohort, which only slightly agreed. The ILAP cohort agreed that the project helped them to make connections between calculus and the "real world", versus the non-ILAP cohort, which only slightly agreed. This result lends some support for the use of ILAPs in Calculus II to enhance student learning.

Fall 2005 Calculus III:

Table 5. Fall 2005 Calculus III; Math-Geosciences ILAP versus Non-ILAP Project:

The fall 2005 Calculus III questionnaire results from the Mathematics-Geosciences ILAP students versus the corresponding results from the non-ILAP students show differences between these two groups with respect to five responses, time spent on the project, number of group meetings, whether the class introduction helped in understanding the project, whether a better understanding of some connections between calculus and the "real world" was gained from the project, and whether students would choose hands-on projects over non hands-on projects.

Over a two week period, the ILAP cohort spent slightly less time on their project, 4 hours (with 3 meetings), than the non-ILAP cohort, which spent 4.5 hours (but with only 2 meetings). The median response values indicate that the ILAP cohort only agreed that the class introduction helped them understand the material, versus the non-ILAP cohort, which strongly agreed. Perhaps without the geoscience interpretation of the mathematics, the class introduction was more important for students in order to understand what to do. The ILAP cohort agreed that the project helped them to make connections between calculus and the "real world", versus the non-ILAP cohort, which disagreed. Finally, the ILAP students agreed that they would choose hands-on projects over non hands-on projects, versus the non-ILAP cohort, which only slightly agreed. These last results lend some support for the use of ILAPs in Calculus III with respect to enhancing student learning.

Fall 2005 Calculus I, II, and III – General discussion of ILAP versus non-ILAP student reaction results:

Some of these Calculus I, II, and III results raise the question of which courses in the mathematics curriculum are better for introducing ILAPs. This depends on the type of students, as well as on other factors. In general, it may be the case that higher level courses are better courses in which to use ILAPs. It may be better for students to focus more on the core topics in the lower level calculus courses and to see ILAPs later in the mathematics curriculum. Additionally, once students have more basic knowledge, they can address more challenging and relevant ILAP topics.

There were some results that the project faculty generally expected, but that were not, in fact, observed. It was expected that the ILAP cohorts would respond somewhat more positively with respect to ILAPs assisting in learning course material, finding ILAPs more interesting, and preferring hands-on types of projects as opposed to projects that were not hands-on. The fact that these results were not obtained more conclusively may reflect specific features of the implementation of this project instead of actual

Status of and Changes in Calculus Students' Conceptions of Mathematics Over Three Semesters (Calculus I Fall 2004 – Calculus III Spring 2005) When Projects Were Used

1 ←	Mean Scores	→ 6
Knowledge as facts, formulas, and algorithms	Before projects: 30.87 After projects: 29.02 t =2.29 p= .027 ← Difference is significant	Knowledge as concepts, principles, and generalizations
Mathematics as a collection of isolated pieces	Before projects: 35.56 After projects: 33.57 t=2.51 p=.016 ← Difference is significant	Mathematics as a coherent system
Mathematics as a static entity	Before projects: 29.42 After projects: 29.96 t = -.552 p= .584 Difference is **not** significant	Mathematics as a dynamic field
Mathematics as results	Before projects: 37.56 After projects: 33.42 t = 5.163 p= .000 ← Difference is significant	Mathematics as sense making
Outside authority	Before projects: 33.49 After projects: 30.84 t = 3.02 p= .004 ← Difference is significant	Logical thought
Learning as memorizing intact knowledge	Before projects: 33.16 After projects: 31.16 t = 2.00 p= .051 Difference is **not** significant	Learning as constructing and understanding
Mathematics as a school subject with little value in everyday life or future work	Before projects: 42.56 After projects: 36.47 t = 4.69 p= .000 ← Difference is significant	Mathematics as a useful endeavor

Note 1: N = 45 Students
Note 2: Total possible score for each of the 7 dimensions was 48 (8 questions x 6 possible points per question)
Note 3: Decision criterion for significance was p less than or equal to 0.05
Note 4: Significant shifts appear in colored font.

Table 6: CONCEPTIONS OF MATHEMATICS INVENTORY (CMI) LONGITUDINAL STUDY

student preferences [7].

Issues that may have impacted students' responses to ILAPs include difficulties due to some faculty not preparing adequately for ILAPs presentations. The ILAPs required significantly more preparation time and effort than did the non-ILAPs. Another issue is that ILAPs may have been viewed by students as "add-ons" to the course syllabus (i.e., interpreted by students as extra work) as opposed to being an integral part of the course. Either of these factors would negatively impact students' responses to ILAPs. A long-term study could provide more definitive results.

V. RESULTS FROM AN ILAP VERSUS NON-ILAP TEST QUESTION CALCULUS II – FALL 2005

In Calculus II, fall 2005, students were given a test question based on the ILAP or non-ILAP (depending on course section). Due to various difficulties, this was the only course in which the same test question was used for both the ILAP and non-ILAP course sections [7]. The same question was used for all sections of this course since the non-ILAP was just a "stripped-down" ILAP (without an interdisciplinary application, etc.). The mean score (out of 10 points) for the ILAP section was 5.8 points versus 6.6 points for the non-ILAP section. These data show non-ILAP students (control group) with better scores than ILAP students – the opposite result from what was expected! The following are some factors to be considered in assessing this result:

(i) The non-ILAP section was later in the day than the ILAP section. In some cases, non-ILAP students may have gotten information about the test question from ILAP students who had already taken the test.

(ii) The entering achievement of the non-ILAP group could have been higher than that of the ILAP group. Intact classes were used because we could not randomly assign students to different calculus course sections.

(iii) Since these results were unexpected, it is important to do follow-up work where careful control of entering achievement can be maintained and the content in the projects is consistently similar between the two groups, ILAP and non-ILAP (except for context).

VI. RESULTS FROM THE STUDENT CONCEPTIONS OF MATHEMATICS INVENTORY (CMI)

At the beginning of Calculus I in the fall 2004 semester, students were given a Conceptions of Mathematics Inventory (CMI) (a questionnaire composed of fifty-six questions) [9, 10]. This inventory was given again in Calculus III at the end of the fall 2005 semester to the same cohort of students, and the two results were compared. The inventory topics consisted of seven dimensions as described below.

Following are the four themes and their related dimensions within the CMI. Students answered questions using a Likert scale from 1 to 6. That is, scoring was 1, 2, 3, 4, 5, or 6, with a continuum of interpretations from 1 to 6. Using the student responses, a mean score was calculated for each of the seven dimensions.

Status of and Changes in Calculus Students' Conceptions of Mathematics During Calculus I Fall 2005 — ILAP Projects Used

1 ←	Mean Scores	→ 6
Knowledge as facts, formulas, and algorithms	Before projects: 29.49 After projects: 30.80 t = -2.53 p= .013 → Difference is significant	Knowledge as concepts, principles, and generalizations
Mathematics as a collection of isolated pieces	Before projects: 28.01 After projects: 27.86 t=.36 p=.719 Difference is not significant	Mathematics as a coherent system
Mathematics as a static entity	Before projects: 30.67 After projects: 31.85 t = -2.56 p= .013 → Difference is significant	Mathematics as a dynamic field
Mathematics as results	Before projects: 31.08 After projects: 31.54 t = -1.05 p= .299 Difference is not significant	Mathematics as sense making
Outside authority	Before projects: 32.65 After projects: 31.24 t = 2.29 p= .025 ← Difference is significant	Logical thought
Learning as memorizing intact knowledge	Before projects: 29.15 After projects: 29.12 t = .067 p= .947 Difference is not significant	Learning as constructing and understanding
Mathematics as a school subject with little value in everyday life or future work	Before projects: 28.46 After projects: 27.74 t = .831 p= .409 Difference is not significant	Mathematics as a useful endeavor

Note 1: N = 78 Students
Note 2: Total possible score for each of the 7 dimensions was 48 (8 questions x 6 possible points per question)
Note 3: Decision criterion for significance was p less than or equal to 0.05
Note 4: 2 ILAPs used
Note 5: Significant shifts appear in colored font.

Table 7: CONCEPTIONS OF MATHEMATICS INVENTORY (CMI)

I. Nature of Mathematical Knowledge

1. Composition of Mathematical Knowledge:
Knowledge as facts, formulas, and algorithms (score=1) versus
Knowledge as concepts, principles, and generalizations (score=6);

2. Structure of Mathematical Knowledge:
Mathematics as a collection of isolated pieces (score=1) versus
Mathematics as a coherent system (score=6);

3. Status of Mathematical Knowledge:
Mathematics as a static entity (score=1) versus
Mathematics as a dynamic field (score=6);

II. Nature of Mathematical Activity

4. Doing Mathematics:
Mathematics as results (score=1) versus
Mathematics as sense-making (score=6);

5. Validating Ideas in Mathematics:
Outside authority (score=1) versus
Logical thought (score=6);

III. 6. Learning Mathematics:

Learning as memorizing intact knowledge (score=1) versus
Learning as constructing and understanding (score=6);

IV. 7. Usefulness of Mathematics:

Mathematics as a school subject with little value in everyday life or future work (score=1) versus
Mathematics as a useful endeavor (score=6);

The results in Table 6 describe a longitudinal study of the CMI results over the three semesters of calculus, from the start of Calculus I in fall 2004 to the end of Calculus III in fall 2005. We did not have a large enough sample size to do a longitudinal study of ILAP versus non-ILAP results over three semesters of calculus. This should be done in future studies. However, the one-semester, fall 2005, CMI study results, presented in Table 7 and Table 8, do distinguish between ILAP versus non-ILAP students. The Tables 7 and 8 results reflect the effect of just one semester of Calculus I with ILAPs (Table 7) or with non-ILAPs (Table 8) on the seven dimensions of the CMI. These students took the CMI at beginning of Calculus I and at the end of Calculus I.

Table 6. Analysis:

The longitudinal CMI study results, presented in Table 6, show some interesting, but disappointing, results. Note that the Table 6 results do not distinguish between ILAP versus non-ILAP students. The results reflect the effect of three semesters of calculus (with either ILAP or non-ILAP projects) on the seven dimensions of the CMI. The data analyses consisted of t-tests for matched pairs (each student's pre-test score is paired with the post-test score). The decision rule for determining statistical significance of a mean difference in scores was p less than or equal to 0.05. The results show that the five (out of a possible seven) statistically significant results for conception shifts, although relatively small, are all in the “wrong” direction! That is, from an educator's point of view, our students show decreases in mathematical maturity/sophistication after exposure to three semesters of calculus. Students' conceptions did NOT move in direction we hoped. Possibly, the projects did not convey what we intended or perhaps there were not enough projects to have an impact. It could be that the general nature of our calculus courses over-rides any effect of projects because traditional calculus courses are so procedurally taught students get wrong impressions of what constitutes mathematics as a discipline.

Tables 7 and 8. Analysis:

The Tables 7 and 8 data analyses were done as preciously described for Table 6. The ILAP data for one semester (Table 7) shows that there were three slight, but statistically significant, shifts from the beginning of the semester to the end of the semester. The corresponding non-ILAP data from the same semester (Table 8) has only one slight, but statistically significant, shift. The results are summarized below, dimension by dimension.

Dimension 1 - Composition of Mathematical Knowledge: The ILAP cohort increasingly thought of mathematics as concepts, principles, and generalizations rather than facts, formulas, and algorithms. This is a generally desirable shift. There was no such statistically significant shift for the non-ILAP cohort.

Dimension 2 - Structure of Mathematical Knowledge: Neither the ILAP nor the non-ILAP cohorts showed a significant shift in this dimension.

Dimension 3 - Status of Mathematical Knowledge: The ILAP cohort increasingly thought of mathematics as a dynamic field rather than a static entity. This is generally desirable. There was no such statistically significant shift for the non-ILAP cohort. Dimension 4 - Doing Mathematics: Neither the ILAP nor the non-ILAP cohorts showed a significant shift in this dimension.

Dimension 5 - Validating Ideas in Mathematics: The ILAP cohort shifted in the direction of thinking of mathematics as a discipline where one relies on an outside authority rather than logical thought to validate results. This shift is not generally desirable but, given the nature of the course, the result may not be surprising. There are a lot of concepts just "given as facts" in a calculus course and this shift may reflect that, rather than the use of ILAP projects. There was no such statistically significant shift for the non-ILAP cohort.

Dimension 6 - Learning Mathematics: The non-ILAP cohort increasingly thought of learning mathematics as constructing and understanding, as opposed to memorizing intact knowledge. This is a desirable shift. Surprisingly, this shift was not manifested in the ILAP cohort.

Dimension 7 - Usefulness of Mathematics: Neither the ILAP nor the non-ILAP cohorts showed a significant shift in this dimension.

The fall 2005 results for the ILAP and non-ILAP sections of calculus I are fairly favorable (Tables 7 and 8). But the results of the longitudinal study are not what was expected or what would generally be considered desirable (Table 6). It is quite probable that the over-all effect of the calculus courses overwhelmed effects of the ILAP versus non-ILAP projects. It may be the case that ILAPs are ineffective when used in small numbers as add-ons to a course. We did not check or adjust for any differences in entering achievement between various sections of thee courses, The reason that we did not do so is that we did not have access to this information, This factor could be taken into account in future studies.

Studies assessing high school students' conceptions of "reform mathematics" did show "positive" shifts in student conceptions when comparing student exposure to "reform" versus "traditional" mathematics curricula [11, 12]. That is, the high school students with "reform mathematics" backgrounds had responses that were more aligned with reform-oriented ideas (what we refer to in this paper as "desirable") than did those students with "traditional mathematics" backgrounds. The high school studies used the same assessment instrument that we did, the CMI, but were longer-term studies (four-year studies). The use of student group projects is one component of "reform calculus". We hypothesize that a project such as the one we implemented, but with more long-term student immersion in calculus and related courses and with ILAPs carefully integrated into the curriculum, would also produce more positive results.

More long-term research is required, especially research that follows a cohort of students who have ILAPs for each of the three consecutive semesters of calculus versus a cohort of students who have non-ILAPs for each of the three consecutive semesters of calculus. We could not do such a study because we did not have control over which students enrolled in

Status of and Changes in Calculus Students' Conceptions of Mathematics During Calculus I Fall 2005 - Non-ILAP Projects Used

1 ⟵	Mean Scores	⟶ 6
Knowledge as facts, formulas, and algorithms	Before projects: **30.21** After projects: **30.95** **t = -1.09 p= .283** Difference is not significant	**Knowledge as concepts, principles, and generalizations**
Mathematics as a collection of isolated pieces	Before projects: **27.18** After projects: **28.39** **t=-1.41 p=.168** Difference is not significant	**Mathematics as a coherent system**
Mathematics as a static entity	Before projects: **31.82** After projects: **31.55** **t = .43 p= .669** Difference is not significant	**Mathematics as a dynamic field**
Mathematics as results	Before projects: **30.13** After projects: **30.82** **t = -.86 p= .398** Difference is not significant	**Mathematics as sense making**
Outside Authority	Before projects: **32.61** After projects: **31.95** **t = .82 p= .418** Difference is not significant	**Logical thought**
Learning as memorizing intact knowledge	Before projects: **28.58** After projects: **30.24** **t = -2.58 p= .014** → Difference is significant	**Learning as constructing and understanding**
Mathematics as a school subject with little value in everyday life or future work	Before projects: **28.21** After projects: **28.95** **t = -1.21 p= .233** Difference is not significant	**Mathematics as a useful endeavor**

Note 1: N = 38 Students
Note 2: Total possible score for each of the 7 dimensions was 48 (8 questions x 6 possible points per question)
Note 3: Decision criterion for significance was p less than or equal to 0.05
Note 4: 2 Non-ILAPs used
Note 5: Significant shifts appear in colored font.

Table 8: CONCEPTIONS OF MATHEMATICS INVENTORY (CMI)

which sections of the courses.

VII. ADVISORY BOARD RECOMMENDATIONS

The project had a national advisory board (see ACKNOWLEDGEMENTS). We used the advisory board recommendations to modify the second year of the project. The following items are some of the suggestions from the advisory board (proposed midway through the first year of the project):

Project Mechanics:

- Consider reusing previous ILAPs (adapt and implement), rather than developing every thing from scratch.
- Give students a selection of ILAP topics.
- Devote more class time to doing group work.
- Provide more structure, and don't hesitate to step students through some requirements with respect to the ILAPs for freshmen (consider small, frequent, straightforward questions). More open-ended, less structured ILAPs can be used for higher-level classes.

Project Management:

- Communicate better to students the value of ILAPs in curriculum, especially with respect to preparing students to function as part of interdisciplinary teams that have to communicate results to others.
- Explain clearly to students that this is a regular part of the course, and that students will do either an ILAP or a non-interdisciplinary, non-applied project.
- Encourage the administration to advertise and support the project.
- Prepare a solid foundation in order to sus tain the project. Make clear to others why ILAPs are indispensable. For exam ple, such projects can be used to en hance Accreditation Board for Engineering and Technology (ABET) and/or North Central Association (NCA) Commission on Accreditation and School Improvement assessments, support interdisciplinary faculty collaboration, and support student learning and integration of curricula.

General Ideas for Enhancing the Project:

- Use some data from departmental labs rather than generating new data for each ILAP. In addition to being efficient, this extends science and engineering labs to other courses in a continuous and interdisciplinary fashion.
- Consider one ILAP per semester rather than two, especially for first semester freshmen.
- Consider ILAP presentations or poster sessions as a change of pace and as a method of developing different modes of communication.
- Consider extending the idea of ILAPs to high school students.

VIII. CONCLUSIONS AND GENERAL CONSENSUS OF PROJECT FACULTY

In this study, we were looking for hypotheses, i.e., trends to be investigated further in larger studies. Earlier papers reporting this project's results appeared in the 2005 and 2006 ASEE Conference Proceedings [7] and supplement the results presented in this paper. Further results have been discussed in Sections IV, V, and VI of this paper. Our preliminary results indicate that just introducing ILAPs into the first three semesters of the calculus sequence, as we did, may not have the desired effect of enhancing students' learning. We have not seen strong significant differences between the data collected from ILAP students versus the data from non-ILAP students. This is likely due, in part, to the use of relatively few ILAPS. It may be that ILAPs are ineffective when used in small numbers as add-ons to a course. Given the fact the syllabi for the first semesters of calculus is packed with topics, ILAPs introduced into the more advanced courses may serve students better than ILAPs in the introductory calculus courses (at institutions similar to ours). However, more study is required, in particular, long-term study.

Based on the two years of experience working with ILAPs and non-ILAPs in the calculus courses, the project senior personnel reached the following consensus:

1. ILAPs may be more useful in upper-level mathematics courses than in the calculus courses.
2. It seems better to focus on the jam-packed calculus syllabi instead of ILAPs.
3. Much time and effort is needed to implement ILAPs well, even if the ILAPs are just modified or adapted from existing ILAPs.
4. More research is needed into the effectiveness of ILAPs in teaching STEM disciplines.

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