What Do Students Perceive During a Lesson on Center-Of-Mass?

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Abstract

This paper presents an assessment of how students perceive a typical classroom representation of center-of-mass. Participants were shown figures consisting of two filled black circles; they were told that these figures represented spheres or balls. They were then asked to indicate with a mark the point where the spheres would balance, i. e., they were asked to find the center-of-mass. There were two participant groups. The first group (labeled "Control") received only brief, written instructions on the exercise. The second group (labeled "CoM") was shown a 15-minute video which discussed, qualitatively, the topic of center of mass and the same written instructions. They also received the same written instructions as the Control group. In all groups students do not estimate the center-of-mass as if they were considering spheres. We find that while the performance for both groups was poor, there is a difference between the two. As our results will show, participants with some exposure to the topic are not influenced by the proximity of the object as are those not so exposed, by a factor of 4 – 6. Consequently, direct instruction can clarify what we call the edge effect, that is, the tendency of participants to locate the center-of-mass of two objects away from either one of them.

I. Introduction

When the concept of center-of-mass is introduced in the classroom, the instructor is likely to draw a shape on the board, or present a shape through projection, and then use that diagram to help define the meaning of center-of-mass. A similar presentation introduces the concept of the center-of-mass of multiple objects. During this introduction the student learns that the center-of-mass can be anywhere; it can be outside of a single object, or inside one of the bodies in a group. Planetary analogies are often made with the latter. But what does the student actually perceive while this discussion is taking place? It is a crucial question; center-of-mass is not only an abstract concept but one for which the student should be able to make a real world connection. By abstracting the images shown during a typical lecture, developing a sense of where the balance point of an object is may not occur. While students may understand the concept of center-of-mass, they may not be able to estimate or perceive where that point actually is. Students don't always understand what is explained during a lecture; the skills they actually employ may not be what were taught. They may instead use their own assumptions or misconceptions to evaluate the center-of-mass.

Naturally, understanding how students learn physics has received some attention (e.g., Arons, 1976; Clement, 1978; McDermott, 1981). Ortiz, Heron, and Schaffer (2005) established that estimating the center-of-mass of irregularly shaped objects is challenging for students, even with direct teaching of the material. Previous work (Author 1 & Author 2, 2001) suggested that students without any teaching would compare the diameters of the circles. One of the goals of this study was to determine if students exposed to the concept of center-of-mass would use the same metric.

The capacity to assess the center-of-mass of an object or group of objects has been well documented (e.g., Bingham & Muchawski, 1993; Morgan and Glennerster 1991). In studies such as these, subjects are presented with numerous "masses" or points, which they must then use to determine the center-of-mass. Often specific symmetry references are included such as triangular or square, that help the subject make the assessment. That research is designed to assess how the subjects determine the center-of-mass. Other studies have examined the developmental aspect of children's ability to estimate how two objects might balance or determine which object on a balance or see-saw is heavier (e.g., Siegler, 1978; Siegler & Vago, 1976). In those studies, the reference of a fulcrum and balance, as well as an indication of weight, was provided to the participants. The stimuli presented gave the participants a clear reference with which to estimate location and relative mass. (The representation of weights were marked and drawn like laboratory scale weights and scales of distance were indicated.) This is in contrast to the current effort, where the participants must choose their own reference point. A study by Ortiz, Heron, and Shaffer (2005), with students that were interested in physics, tested a similar demographic to the current effort. In that study, images of actual objects (such as baseball bats) were presented. In that paper it was noted that students had difficulty in locating the actual center-of-mass, even after instruction was provided.

Authors 1 and 2 (2001) examined the ability of subjects to assess center-of-mass. In that effort marks, measures, and direct indication of relative mass were not provided (other than for an example in the instructions, as seen in Appendix 1). Instead, the objects under consideration were represented abstractly, as might actually be presented in a classroom on a blackboard. That is to say, two darkened circles were presented and the subjects were instructed to view them as planets or balls. While representational images (such as see-saws, ball bats, balance scales, etc) are often used to teach the concept of center-of-mass, they are not used exclusively (for example the center-of-mass of the earth-moon system would not typically be placed on a fulcrum). When presenting a written or printed image, the mass of the objects must often be inferred from their size, and the point in space between the two objects has no graph lines or marks for guidance. The methods of the current investigation are identical to Author 1 and Author 2 (2001).

The results of Author 1 and Author 2 (2001) had implications for teaching the subject of center-of-mass. Specifically, the question was, "how would the results change with direct instruction?" Thus, in the current investigation some of the subjects were provided with background material on center-of-mass, as described below. The remaining subjects were not given any background; they were the control group.

II. METHODOLOGY

IIa. The Groups

The participants in this study were all college students. In a Author 1 and Author 2 (2001) it was found that gender had no effect on results; previous physics experience had only a marginal effect. The participants were divided into two groups. The first (control) group, called "No Video," received only the questionnaire. The "No Video" group allowed the investigators to test the native ability of the participants to determine the center-of-mass of two representational objects. There were 90 participants in this group; 40 males and 50 females. The median age was 19 with a standard deviation of 1.0. As can be seen in Figure 1, the participants were given pairs of black circular dots to assess. The task of the participants was to determine the center-of-mass of each pair.

The second group, labeled "CoM," was shown the video on center-of-mass. There were 43 participants in this group; 11 males and 32 females. The median age was again 19 with a standard deviation of 1.0. The investigators' intent here was to assess whether instruction on the topic would improve the participants ability to estimate the center-of-mass of the two bodies. Ratio and Separation Distance have the same meaning as before. The groups were similar in all other respects. In both cases, a small number of estimations were rejected due to recording error by the analyst or unclear marking by the participant. Due to their small number these rejections did not alter the results.

IIb. THE QUESTIONNAIRE

The questionnaires were printed on standard (8 ½" by 11") copy paper and held together with one staple. In the written instructions, the participants were asked to interpret the two blackened circles as spheres and estimate the balance point between them. The instructions were as follows:

"Your task is to judge the balance point between the dots in each pair. The balance point is a position of equilibrium. One way to think of this point is to imagine the two black dots on either end of a see-saw. The balance point would then be the location of the fulcrum which would balance the see-saw so that it does not tip one way or the other. The balance point can be located anywhere between the centers of the two dots."

No instruction about calculations or measurements was provided. The full instructions are presented in Appendix 1. The participants were tested on their immediate response, not on an ability to make detailed calculations. The subjects were told that the blackened circles in the questionnaire were spheres and they were instructed to mark the center-of-mass, or "balance point" between the spheres. The proctor was available to clarify the instructions but never added any additional information or direction. The rest of the pages were similar to Figure 1, which is an actual page from the questionnaire. The filled-in circles, representing spheres, were produced in three sizes. In terms of the smallest diameter circle, they have radii of 1, 2, and 4. Thus each pair of circles can be in a ratio of 1:1, 2:1, or 4:1. All three sizes are shown in Figure 1. The participants made their pencil marks directly on the paper. These marks were then later measured with respect to the edge of the larger circle (or left circle if they were the same size). These raw distances were later converted to the same reference: the center of the larger circle (or exactly midway if the circles were the same size). The estimation was then compared to the position of the actual center-of-mass.

No measurement tools or calculators were made available, just a pencil to mark the estimated the center-of-mass. The participants had approximately 25 minutes to complete the task. The questionnaires were distributed to groups of 20 – 25 individuals at a time. Some of the smaller groups were shown the video, some were not. Participants did not choose their group.

The Questionnaire was developed for a previous study by Author 1 and 2 (2001). The format of the Questionnaire was designed to represent the use of circles to represent spheres on the printed page or on a blackboard. The specific ratios and separation distances were determined by several factors. These included the restrictions of page size, minimizing the influence of one pair upon another, and keep the pair within the field of view of the participants. Further consideration was given to the actual location of the center-of-mass with respect to the dimension considered, i. e., diameter, area, or volume. The latter would have to be inferred.

IIc. THE VIDEO

The video was produced entirely in-house; the author who gave both lectures was responsible for content and ensuring that the pace and style of presentation was similar. The commentary was scripted in advance; there was no "ad-libbing." There was no accompanying music, soundtrack, or animation. The presentation was moderately paced, the language was typical of an introductory conceptual lecture, and the enunciation was clear. To provide additional illustration, students not participating in the study were recruited to provide demonstrations and additional explanation of the concepts in the video. Demonstrations were done inside a freshman laboratory. The video avoided mathematics and direct instructions on calculations; rather they qualitatively explored the topic through discussion and demonstration. In short, the video was constructed to be a short version of a typical lecture that a student could experience in a standard college classroom environment, sans calculations.

The center-of-mass video featured demonstrations using balls of different shapes and sizes, people balancing in different positions, a twirling baton, and a geometrical demonstration using rectangular shapes against a grid background. Most of the explanation was provided by the instructing professor; some was done by the recruited students as they performed the demonstrations. A brief, but distinct, comment informed viewers that that center-of-mass could be anywhere, even inside or outside of the bodies under consideration. Subjects were also informed that the center-of-mass of two bodies could be found by taking the ratio of their masses.

The professor, students, and editing team were not informed of the purpose of the video prior to production to prevent any bias in their presentation. They were instructed to develop the video as if they were to be used as a teaching aid.

IId. ASSESSMENT OF PERFORMANCE

To understand how the participants compared the two filled circles, their choices were compared to the actual center-of-mass. Center-of-mass is dimensional, so the question is: did they take the ratio of the diameters, the areas, or did they compare the volumes of the spheres? The actual center-of-mass was calculated for each case using the formulas presented in Appendix 2. That value was compared to the participant's estimation via the percent or fractional error (FE).

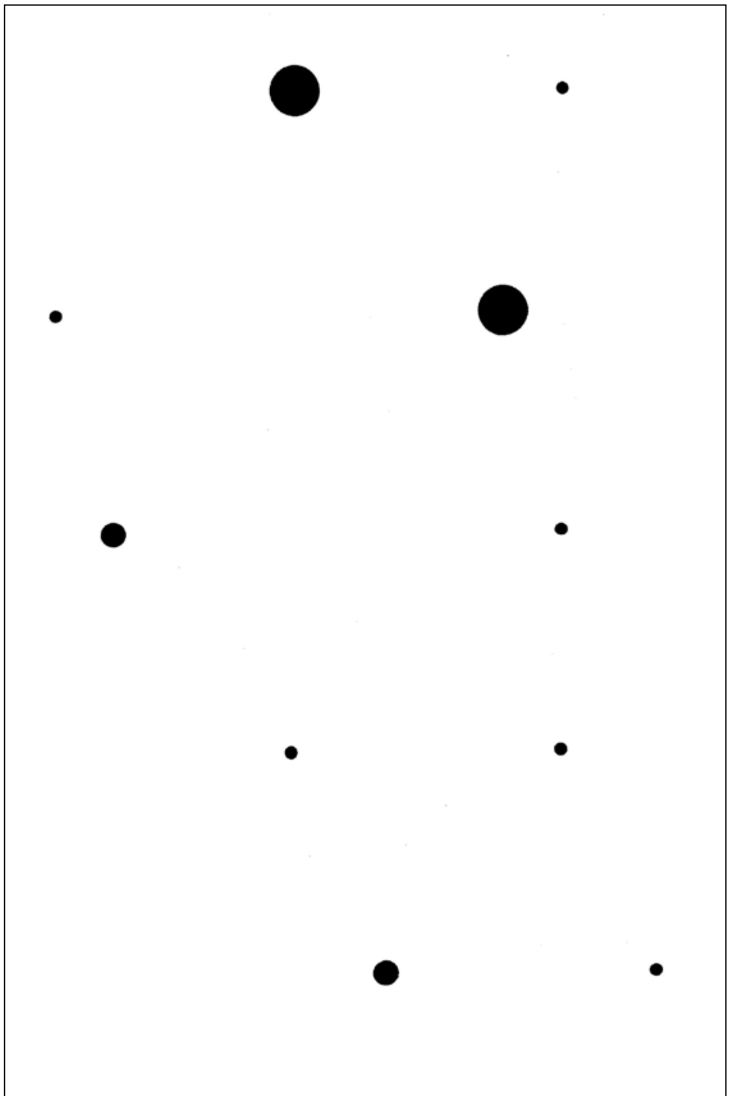

Figure 1

Equation 1 gives the formula for this metric.

$$FE = \left| \frac{COM_{actual} - COM_{estimated}}{COM_{actual}} \right| \quad (1)$$

As can be seen, the larger the difference between the estimated and actual values, the larger the FE will be. By plotting this value vs. the parameter of interest a measure of how well the participants performed is obtained. This is discussed in the results section.

III. RESULTS

The two variables in the study were the ratio of the size of the dots in each pair (called Ratio) and the distance between the dot centers (called Separation Distance). For the "No Video" control group, there was a significant main effect of Ratio, F(2, 801) = 379.1, $p < 0.1$. There was a significant main effect of Separation Distance, F(2, 801) = 4,153.0, $p < 0.1$. The interaction between these two variables (Ratio and Separation Distance) was also significant, F(4, 801) = 228.0, $p < 0.1$.

For the "CoM" group, there was a significant main effect of Ratio, F(2, 378) = 579.1, $p < 0.1$. There was a significant main effect of Separation Distance, F(2, 378) = 2,392.1, $p < 0.1$. The interaction between these two variables (Ratio and Separation Distance) was also significant, F(4, 378) = 1116.0, $p < 0.1$. If the subjects employed quantitative methods to determine the center-of-mass, such instruction should have improved their results. If, on the other hand, the assessment of center-of-mass is purely perceptual, no difference would be expected. Such a determination is an important consideration for teaching the subject.

Some estimations were eliminated from the study. These included cases with no marks of indication, unclear or multiple marks, or marks made outside of, rather than in-between, the two darkened circles.

In Figure 2a and 2b we examine the center-of-mass evaluation, using the case of 4:1 radii ratios. The results strongly suggest that for both the No Video group and the CoM group the participants are using the diameters to estimate center-of-mass. The fractional error is an order of magnitude less as the assumption goes from sphere to area to diameter. The 4:1 radii were used because they most clearly demonstrate the effect.

Of course, if the participants had a very poor sense of scale their estimations would naturally be inaccurate. In a previous study (Author 1 & 2, 2001) it was shown that the participants were able to estimate size and distance well. Moreover, if it was only their scale that was off one would not expect to see such large differences for dimensionality. What is interesting is that it is the diameter that is being used. While the volume would have to be inferred, the area of the circles is clear. Thus one might initially assume that area would be the choice of the participants' comparison; this was not the case.

In Figure 3a, the case of spheres with radii ratios of 1:1 is shown. The error bars represent the estimation error of the participants which was the largest error affecting all of the plots. The results for both the No Video and CoM conditions are virtually the same.

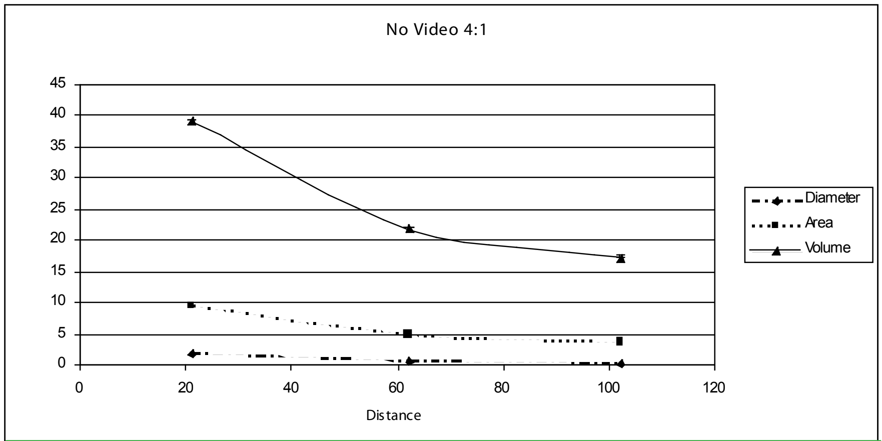

Figure 2A

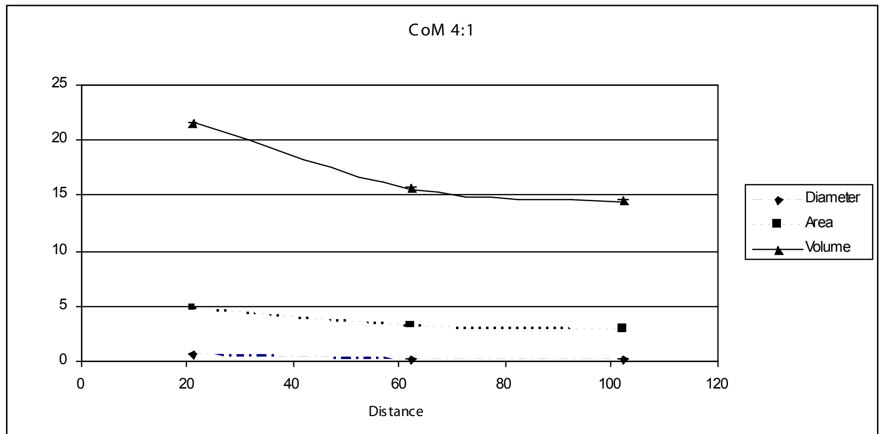

Figure 2B

A slight difference, just outside statistical error, shows that for small separation distances the CoM group did marginally better. The poor result appears to be the influence of the proximity of the edge of the spheres, which is seen more clearly for the case of 2:1 radii ratios (Figure 2b). Author 1 and Author 2 (2001) observed that participants did not mark the center-of-mass near the edge or inside the darkened circle, even when that was the correct position. This was termed the edge effect. The true center-of-mass for two symmetrical objects like spheres or circles will lie somewhere between the object centers; moreover it can lie anywhere between the two object centers, including inside of one of the objects. In general, the actual center-of-mass will like closer to center of the larger object. In the case of the sun and earth system, the center-of-mass lies almost at the sun's center. The results of Author 1 and Author 2 (2001) show that the participants' error in estimation increases the closer the true center-of-mass is to the center of the larger object. The edge effect was dramatic and pervasive throughout all cases. This was still true for participants in the current study who viewed the COM video, but the error was reduced. It was explicitly stated in the CoM video that the center-of-mass could be inside one of the spheres, and it was found that participants in that group performed better in this respect.

When the radii ratio is increased to 4:1 the results are more striking. The No Video control group underperforms for any separation and the edge effect is much greater for that group, as well. Still, the actual performance is poor for both groups. Error bars for Figure 3c are smaller that the points marking the Fractional Error (FE).

IV. CONCLUSIONS

Given the constraints of this study, where participants did not use calculation to estimate the center-of-mass of a two body system, it is observed that a comparison of diameters is used. Despite this simple approach to visualizing the two black dots, the fractional error is poor. The understanding of three dimensions is lost, even though this was directly indicated to the participants in the study. The lack of perception of three dimensions represents a significant challenge to teaching center-of-mass. When participants are given a brief, qualitative lesson on center-of-mass, their performance improves slightly. This is true for both accuracy when compared to the actual location of the center-of-mass and for the resistance to place the center-of-mass near the boundary of one of the black dots (the so-called *edge effect*).

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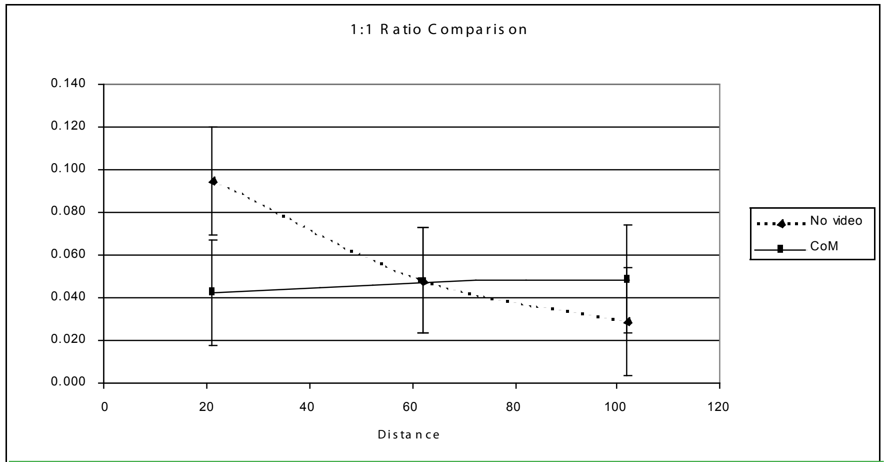

Figure 3A

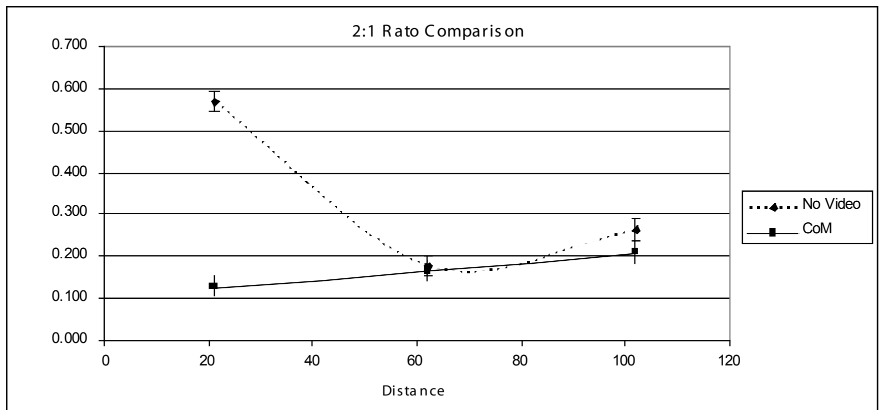

Figure 3B

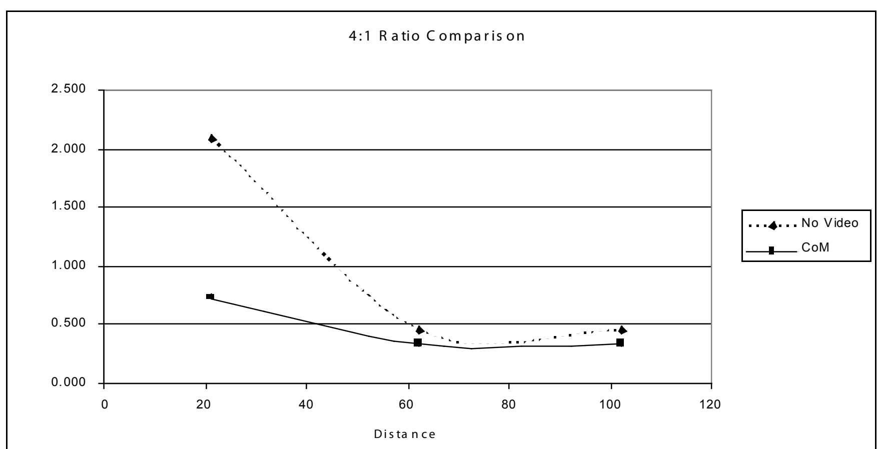

Figure 3C

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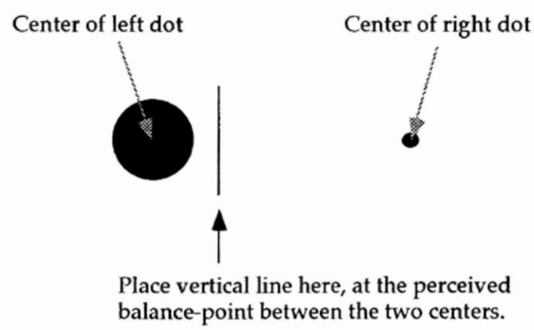
Appendix 1

Experiment Instructions

Manhattan College / College of Mt. St. Vincent
Department of Psychology
Experiment Instructions

In this experiment, you will be presented with pairs of filled black dots. The dot-pairs are located horizontally apart from each other, one on the left, the other on the right. Sometimes the dots in a pair will be the same size and sometimes one will be larger than the other. They are offset to the left and right from the center of the page and so do not line up with each other.

Your task is to judge the balance-point between the dots in each pair. The balance-point is a position of equilibrium. One way to think of this point is to imagine the two black dots on either end of a see-saw. The balance-point would then be the location of the fulcrum which would balance the see-saw so that it does not tip one way or the other. The balance-point can be located anywhere between the centers of the two dots. Record your judgement by drawing a vertical line where you think it is.



Turn the page and begin only when you are told. Make sure that the page is oriented straight up and down. Do not tilt any of the pages. When you are done with one page you may proceed to the next.

When you have finished estimating the position of the balance-points, you can begin the next section of the experiment. Read and follow the directions for each section as indicated.

If these instructions are not clear, or if you have any other questions, please ask the researcher. If you make a mistake, or wish to redo a judgement, tell the researcher.

Appendix 2

Determining the center-of-mass of two objects.

The center-of-mass of two objects in one dimension can be found by the following equation:

$$X_{COM} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2} \qquad \text{(A1)}$$

where X COM is the center-of-mass (CoM) relative to an arbitrary reference point, X_1 the distance to the first object, m_1 the mass of that object, X_2 the distance to the second object, and m_2 the mass of that object. Equation A1 can be found in any Introductory Physics text such as Halliday, Resnick, and Walker (2003). If the masses are of the same uniform density the above equation can be re-written in terms of the volumes. In our study we make that assumption. Consequently, we obtain the following by substituting into Equation (A1):

$$X_{COM} = \frac{\rho V_1 X_1 + \rho V_2 X_2}{\rho V_1 + \rho V_2} \quad . \qquad \text{(A2)}$$

where ρ is the density of the spheres, and V_1 and V_2 the volumes of the first and second sphere, respectively. Since the volumes are proportional to the radii cubed of the spheres, the Equation (A2) reduces to:

$$X_{COM} = \frac{R_1^3 X_1 + R_2^3 X_2}{R_1^3 + R_2^3} \quad . \qquad \text{(A3)}$$

Finally, if the distances are measured from the center of the sphere with the larger radius, X_1 equals zero and we obtain:

$$X_{COM} = \frac{X_2}{1 + \frac{R_1^3}{R_2^3}} \quad . \qquad \text{(A4)}$$

Equation A4 also provides a way of interpreting the results of this study. When densities are equal the true center-of-mass depends only on the relative volumes of the spheres. But it is also possible to define a two-dimensional, as well as a one-dimensional, center-of-mass. When the participants in this study estimate the center-of-mass they may be imagining the darkened circles as spheres, or they may be comparing the areas of the observed circles, or they may even be using the diameters as their metric of size. If Equation (A4) equation is written as

$$X_{COM} = \frac{X_2}{1 + \frac{R_1^N}{R_2^N}} \quad , \qquad \text{(A5)}$$

where N is the dimension and can have a value of 1, 2, or 3, then the participants results can be compared to the actual results for volume, area, or diameter.