Using Mathematics and Engineering to Solve Problems in Secondary Level Biology

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There are strong classroom ties between mathematics and the sciences of physics and chemistry, but those ties seem weaker between mathematics and biology. Practicing biologists realize both that there are interesting mathematics problems in biology, and that viewing classroom biology in the context of another discipline could support students' development of biology understanding (as mathematics does for physics and chemistry). The Biology Levers Out Of Mathematics study, implemented in public and private schools throughout a metropolitan area in the northeastern United States, tackles this challenge by introducing engineering as a bridge connecting the heretofore isolated silos of classroom mathematics and biology. This study presents engineering design methods for students to use in the posing of biology problems that mathematics then makes possible to resolve. Interviews with teachers and observations of students suggest that this approach makes the understanding of inheritance processes accessible to a wide range of the study's participants.

Habits of Mind Could be Socialized in Biology Classrooms (But Aren't)

Secondary level biology does not offer students many opportunities to deal with real-world problems along the lines that actual biologists would address in their day-to-day professional practice. This is somewhat negligent on the part of curriculum designers and textbook authors when these problems can be genuinely intriguing and therefore engaging to students. For example, breeding an endangered species in captivity takes on a whole new meaning when that species happens to be a tiger.

But engagement can also arise from students being able to sympathize with the stakeholders in problems (Rosson & Carroll, 2009). Entries to complex problems will appear for students who recognize people in the problems just like they are, characters who resemble members in students' families or communities, doing things just as the students would, and facing a situation that biology expertise will help them resolve. Or, with the addition of some desirable difficulty, entries to problems can occur where the expertise required is in the students' zone of proximal development (Vygotsky, 1978), just beyond what they already know but yet can accumulate in the process of addressing the problem. The contextual change involved is neither abrupt nor uncomfortably immense, involving a move from the accustomed status of being students in order to try on the roles of consulting professionals and their clients. Such a change might be as simple as consciously leaving behind the role of bewildered students.

Instead, secondary level biology students are often handed a sequence of well-defined concepts (e.g., DNA, genes, chromosomes) associated with well-defined relationships and processes (e.g., transcription, dominance, random assortment). No one can see or watch these components without microscopes, and they remain abstract throughout students' association with them. Meanwhile, the same students encounter similarly well-defined abstractions in their mathematics courses, mathematics that could be applied to those biology processes (the way that engineers apply mathematics principles to resolve physics and chemistry problems) in order to demonstrate, explain, predict, and even influence those processes. Except that currently such mathematics is not applied, so those opportunities to move beyond abstraction are wasted.

Skills and Concepts in Biology Leveraged through Mathematics and Engineering

The Partnership for 21st Century Skills groups learning and innovation skills under the mnemonic of four Cs: critical thinking/problem solving, communication, collaboration, and creativity/innovation (2011). When the scientific work at hand is "building and refining models of the world" (Lehrer, Schauble, & Lucas, 2008), each of these skills can be addressed in biology in the manner that engineers use when approaching a problem: first in the expression of a student's initial model (e.g., of a biological process), then in the testing of that model, followed by the modification of that model until it is found to satisfice. Consistent and legible expression of a model framed as mathematical algorithm facilitates peer collaboration.

It must be noted that such an engineering design approach need neither supplant nor conflate biological processes with mathematical processes. Rather, this approach introduces opportunities for both convergent (increasingly exclusive deduction) and divergent (increasingly inclusive induction) investigations and conclusions (Dym, Agogino, Eris, Frey, & Leifer, 2005) as information about a biological process is uncovered and subjected to scrutiny. Using mathematics as a vehicle for transfer from simple examples to complex projects also incorporates transfer across otherwise discrete domains (i.e., mathematics and biology), enabling expression in terms that can be shared among fellow students as researchers (Lobato, 2012).

The positing of an engineering problem based on a biological process is not difficult. Consider Mendel's Law of Segregation of Alleles in the case of an animal whose genes each have two alleles, and for which each allele may be either type *A* or type *a*. Each parent could then be one of these genotypes: *AA*, *Aa*, or *aa*. Say the male parent is *Aa*, and so is the female parent. When a gamete from the male parent and a gamete from the female parent fuse as a zygote, there is no way to predict which of the two alleles from the male parent (hereinafter referred to by the term "dad"), *A* or *a*, is present because both are equally likely. This is also the case for the allele contributed by the female parent (or "mom").

However, when applying these principles as an engineer might, one can predict the range of possible outcomes for an offspring having those parents. Likewise, one can determine which of those outcomes (*AA, Aa, or aa*) is more likely to occur than others through the translation into a mathematical algorithm whereby *A* from mom and *a* from dad is seen to be the same as *a* from mom and *A* from dad, and thus there are likely to be twice as many *Aa* offspring as either *AA* or *aa*. One trick in applying mathematics to biology is to establish and maintain sensible mapping of mathematical expressions onto biological phenomena (e.g., irrational numbers might not mean much when dealing with alleles). And in this case, Mendel's Law of Assortment of Alleles can be handled by extending the algorithm from the Law of Segregation of Alleles to multiple genes.

The Structure of *Biology Levers Out Of Mathematics*

Biology might not be the first science that comes to mind when mathematics-science or engineering-science integration is mentioned, but there are plenty of opportunities to:

- frame biology problems to require engineering investigations with multiple resolutions
- enhance students' understanding of biological processes during the investigation, leading to a variety of expressions including mathematical ones as necessary to communicate findings
- modify the context of an investigation so students extend themselves beyond the classroom

The Biology Levers Out Of Mathematics (BLOOM) study takes advantage of those opportunities that have until now lain dormant. To provide a model for others to build upon, we discuss in depth an example of the BLOOM modules that have been developed and implemented, as well as the effects on students that we have observed. The duration of a module can vary between two to four weeks of daily classroom 45-minute sessions, during which time some mathematical expression can be derived that describes a biological process in algebraic terms. This expression may be part of a more complex algorithm, but the variables should never be dissociated from the biological phenomena under consideration (i.e., no irrational alleles).

What students encounter in BLOOM are the sorts of situations that engineers face:

- determining preferred conditions in comparison to existing conditions
- defining a problem from the gap between existing and preferred conditions
- analyzing the needs of the parties involved in that problem
- matching those needs with the resources available
- deriving a plan to get from the existing conditions to the preferred ones

For a BLOOM module, students perform a simplified version of engineering design in order to define and achieve a goal, using what they know or can find out from biology and mathematics content, perhaps inventing or modifying technologies as they need them. For example, we will discuss the unwieldy technology of a Punnett square in this implementation, and how mathematical reasoning supplants the Punnett square both in utility of calculation and fidelity of representation regarding the associated biological processes.

In moving beyond the classroom context, students should report to a fictional client who presents them with a well-defined but ill-structured problem (Simon, 1977), including plausible constraints such as a budget. Being ill-structured, the problem has the appearance of being wicked (Rittel & Webber, 1973), a departure from typical problem solving for most students, and an entry into an engineering design processes. "The wicked-problems approach suggests that there is a fundamental indeterminacy in all but the most trivial design problems," according to Buchanan (1992). And there are indeterminacies associated with not only the formulation of the problem to be resolved in the BLOOM module, but also with the nature of the solution space, and further with the process of getting from the formulated version of a problem to a resolution of that problem.

That noted, the problem is more well-defined than it at first appears, functioning less wickedly than what engineers potentially encounter and more tamely in the manner of a puzzle. There are several ways for the pieces of such a puzzle to be assembled, but only from a finite range of those pieces and with respect to established beginning and end states.

One opportunity provided by the problem's ill-structure is for students to choose from a variety of lenses through which to view their client's problem (different aspects are important to different stakeholders) and proceed to a level of complexity in their resolution which could challenge them but does not have to overwhelm them. Say that the client is looking for a technological resolution to his or her need. Depending on the perspective that a student takes, that technology could be any concept, method, material, or tool that satisficed the client's need.

Of course, the subject matter for our modules has to correlate with applicable standardized tests. Otherwise, there is little incentive for teachers to let us into their classrooms. In our case, we consult the existing Pennsylvania System of School Assessment tests for mathematics and science in 8th and11th grades (Pennsylvania Department of Education, 2012a), the soon-to-be-implemented Keystone Exams (Pennsylvania Department of Education, 2012b) that was to occur at the end of algebra and biology courses, and the impending Common Core State Standards Initiative (2011) and Next Generation Science Standards (2012), that will be factors with regard to what mathematics and biology (and engineering) topics that assessments will target in the near future. We also monitored topics, items, and results from the National Assessment of Educational Progress (Institute of Education Sciences, 2012), the Office for Economic Cooperation and Development's Program for International Student Assessment (2012), and the National Research Council's Framework for K-12 Science Education (2012).

To summarize the ongoing iteration as a research design that informs the development of a BLOOM module, we had to:

- identify topics of interest common to most biology curricula and standardized testing content (e.g., inheritance or evolution).
- identify processes treated within those topics that could be described mathematically (e.g., probabilities of offspring genotypes or rates of change in allele ratios in a population).
- construct an experimental instructional module and associated materials that present students with a problem involving a biological process, the engineering of both a resolution to the problem and the plan to achieve that resolution, and the manner to find mathematical information that they will need to prove that their plan works.
- provide, as part of the module, opportunities for students to explicate their personal models of these processes in the company of their peers (Lesh, Hoover, Hole, Kelly, & Post, 2000)

and to develop mathematical expressions for the processes by modifying their initial models with input from others, thereby performing an iterative process in order to address the goal.

- as part of the module, describe those opportunities such that students would find them plausible to perform, such as buying, breeding, and selling small animals in order (Rosson & Carroll, 2009) to achieve the goal.
- develop and carry out professional development sessions for participating teachers to understand and familiarize themselves with the intended implementation of materials in the module (e.g., what sequence aspects of students' models of the biological process will be sought in order for students to analyze and refine their models).
- administer pre-tests for attitudes of students regarding mathematics use in biology.
- once the module begins, perform daily observation of participating teachers implementing the module in their classrooms as well as daily interviews with those teachers at the end of each class session (as we did for all teachers in all implementations).
- assess attitudinal changes in students not only from pre-test to post-test, but also from interview and observational data, and compare participating teachers' assessments of students' content knowledge with those of students not exposed to the experimental module.
- analyze the successes and blind alleys in each version of the implementation in order to improve the module for presentation at the next professional development round with either new or repeat participating teachers.

Keeping that in mind, readers are urged to consider the in-depth curriculum description that follows as a model of how this kind of mathematical integration and engineering design crosscutting can happen in biology. In other words, we present a reflection on critical elements that supported our outcomes rather than a typical methods and results section.

Curricular Results: Iteratively Prototyping Structure of a BLOOM Module

One module we've implemented (four iterations in a succession of rapid prototyping with from three to six teachers each time) has to do with the biological topics of genotypes, phenotypes, and inheritance. There were six different schools involved altogether, urban public and parochial. The ages of participants ranged from middle school to high school (14-18), and there were about 30 students in each class for an approximate total $N = 180$ during the last round before this report was prepared. All the courses addressed introductory life science, and no background was expected from students in the domain of biology, on the topic of inheritance, or on the topic of genes. The duration of the module's implementation varied from four to six weeks, depending on how often the classes met and how rapidly the teachers were comfortable with moving through the phases.

Typically, inheritance is taught with a focus on the narrative of meiosis, and expected ratios of offspring genotypes are calculated via the Punnett square. But the Punnett square is a clumsy process that rapidly submerges biological meaning in exponential complexity. If students were given another way to look at inheritance might it deepen their understanding of the process? If we scaffolded students quantifying the mechanisms that govern genotype and phenotype, would they learn the biology more deeply?

In this novel module, we created opportunities to help students explore the underlying model of inheritance via multiple representations, including, verbal (e.g., generating rules for inheritance and justifying their claims), pictorial, and mathematical (e.g., tables, equations). The ill-structured well-defined premise involves having a zoo for a client, and this zoo wants some rare animals as a draw. The zoo's animal search committee has approached professional breeders, but found their methods too unpredictable to risk funding. So now the zoo wants a specific plan for breeding animals, with the provision that the animals they get are not only rare, but also pure-bred so that the zoo can continue to breed similar offspring. And when we put students in the context of consulting experts in biology, we have a plausible reason for the zoo to approach the students with its problem and associated constraints.

Thus we had framed a biology problem as a design challenge (see below) for students to pursue via an engineering investigation in order to provide the technology of a breeding plan to the zoo. What we intended as the vehicle for allowing multiple resolutions was the purposeful ambiguity of what makes an animal rare. This was one way for a variety of students to gain entry to the problem, each having a context of expertise

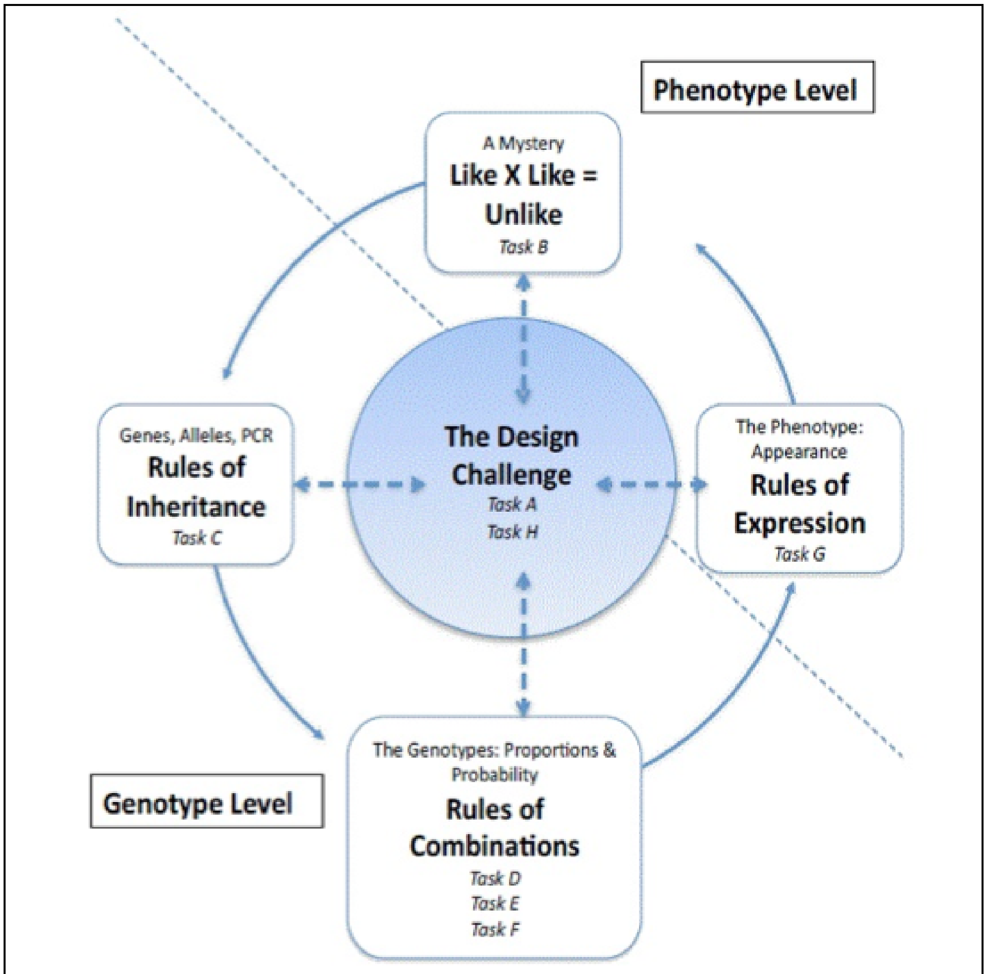

Figure 1. Unit flow for curriculum vision, sequentially from Tasks A to H (note that at the commencement of the module's implementation, students were not expected to have prior knowledge regarding any of the topics)

and direction. For example, the breeders whom the zoo turned down had a definition of rarity based on the time in generations of geckos and effort required for them to find the cross of parents that produced an offspring with some aggregation of desirable traits. The systematicity of their methods was questionable, but they could recognize and testify to an offspring's rarity in an ordinal manner, ranking the results by what they think it would cost to accomplish each one.

Then there was the association of rarity with the comparative expense of acquiring an existing gecko and its traits (such a scale of gecko prices was provided to students), earmarking certain of those traits as already being rare and anticipating novel aggregations of those traits to be even more so. Given six traits, we specified that the zoo would accept at minimum the expression of two, but we also left it open for those students wishing to attempt expressions of from three to all six.

A third interpretation quantified rarity by mathematically determining ratios of allele permutations. When considering the range (i.e., kinds) of genotypes, one sees that origin of an allele has no bearing: there are only three possible permutations (*AA*, *Aa*, and *aa*). However, when considering the likely ratio of one genotype to another, one sees that the origin of an allele is crucial because *Aa* can result both from a combination of dad's a and mom's *A* and from a combination of dad's *A* and mom's *a*, making it twice as likely as either *aa* or *AA*. Clearly, an algorithm composed of mathematical expressions, distinctly describing the predictable contribution to inheritance from each parent's set of alleles (both in range of resulting kinds and relative amount of each kind), can lead to a quantification of rarity.

Curriculum vision. We chose to background the process of meiosis (the steps that result in gametes), opting to foreground the variety of possible results, that is, what permutations of alleles can be passed on to offspring as genotypes, what combinations of alleles predict about the likelihood of a given genotype, and what the dominant and recessive relationships of those genotypes express as phenotypes and in what ratio. Although traditional instruction explores inheritance at the phenotype level in order to describe and infer what is going on at the genotype level, roughly 2/3 of the instructional time, or about four weeks out of a maximum of six, in our unit is focused exclusively at the genotype level in parents and offspring (Tasks C through F, "The Rules of Inheritance"

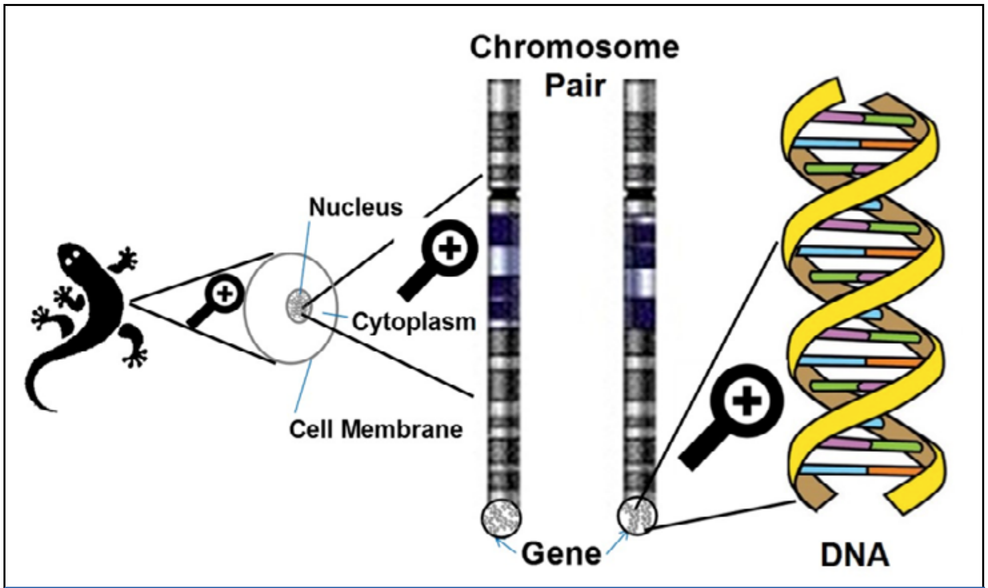

Figure 2. Organizational zoom highlights each level as it is entered

and the "The Rules of Combinations" phases as indicated in Figure 1). We posit that this direct focus on genotype is not only more representative of modern science, but also presents a less demanding way to engage students in mathematical reasoning around the allele mechanisms, separated from the extraneous cognitive load and confusion with phenotype (Sweller, 2011).

Drake and Sherin (2009) refer to curriculum as a vision of the particular kinds of learning and teaching practices described in the curriculum materials (p. 324). The flow in Figure 1 represents the module's curriculum vision, composed of an initial exposure to the design challenge (say one class period or less than a week), four instructional phases ("The Mystery" phase as the remainder of the first week, "The Rules of Inheritance" and the "Rules of Combination" phases as a little under four weeks total, "The Rules of Expression" phase as the remainder of the fifth week and the beginning of the sixth, and the final design challenge as the concluding week). Of course, teachers stretched or compressed their implementations as they determined to best meet the needs of their students.

Mathematics implicitly resides in each phase and engineering design is at the core of each phase, although not always the primary emphasis. This section will highlight the intent of the learning at each phase and the affordances this flow offers with respect to the mathematics, biology and engineering practices.

The design challenge. How do we know what offspring will look like, or how many will look a certain way? Can we predict this? What is rarity as it relates to inheritance? One of our goals is to move students from a qualitative explanation of inheritance to a quantitative model that can be used to make biologically grounded predictions about inheritance. So at the heart of what students are doing is integration of biology and mathematics in order to develop a scientific explanation as a model for the zoo of how they could breed any rare species. At each stage, students question how the new information they encounter informs the design choices they make.

Phase 1: A discrepant event where like × like = unlike. Motivated by their own need to know, students are presented with two contrasting cases. On the surface, these two cases should apparently produce the same results, but they do not. Two identical looking female normal geckos are each mated with the same blizzard male gecko. Students are asked to predict the outcome of the two matings (i.e., crosses), and are then shown that the actual results for one mating are different from that of the other. "How can the same looking parents produce different offspring?" arises as the question of interest.

Students believe genetics is unpredictable or have the idea that a parent can give a lesser amount of a trait to an offspring. They also intuitively know that it all depends on the genes and that sometimes "genes hide" and appear in later generations. This gives us the entry to do a "deep dive" into the genes of the gecko. Rather than consider all the genes a gecko has we agree to look at one gene to see if we can make sense of how genes work. Maneuvering among scales is scaffolded by an organizational zoom (see Figure 2), highlighting the level being emphasized.

Phase 2: Rules of inheritance: PCR, genes, alleles. Leaving the organism level of phenotype, we delve at the genotype level. Utilizing polymerase chain reaction (PCR, see representation in Figure 3), a technique that scientists use to see what is happening in an organism at the DNA level, students make sense of variant forms of a gene (its alleles). This provides evidence from which students infer rules that account for the parental and offspring "footprint" on the printout.

This phase engages students in mechanistic thinking in that it addresses the idea that you can get only one "whole thing" from mom and "one whole thing" from dad (i.e., half of the genetic information an offspring has for a particular trait comes from mom and half comes from dad). This clarifies the conception that students have about a parent's ability to pass a lesser or greater amount of a gene. By showing that parents pass on a whole set of alleles, and that half of what the offspring has comes from mom and the other half from dad, it highlights and clarifies that the whole is a fusion of equal contributions.

Students do not use the canonical vocabulary at first. They describe what they observe as bands or lines and once they have conceptually accounted for the rules, they label what they see as alleles, or forms of a gene. Later, they will use models of the alleles to explore how those behave. Additionally, the PCR being provided shows 12 offspring instead of the 16 that the geckos can have (one of the design constraints). This requires the student to engage in proportional reasoning to decide what alleles the other offspring might inherit from the parents. They also can begin to quantify what percentage of the offspring has a particular genotype.

These kinds of open-ended instructional tasks give multiple entry points for students to begin to make sense of the PCR data as related to inheritance. But interpretation of PCR data is only one example of how students are encouraged to ground their observations in mathematical expressions, at first through a crude manipulation of materials that leads to a recognizable consistency of results, and then through increasingly compact and sophisticated representations (e.g., drawings, tables, and graphs) that lead to robust models of the biological processes under scrutiny.

Phase 3: Rules of combination. High school-aged students come to the class knowing that babies come from

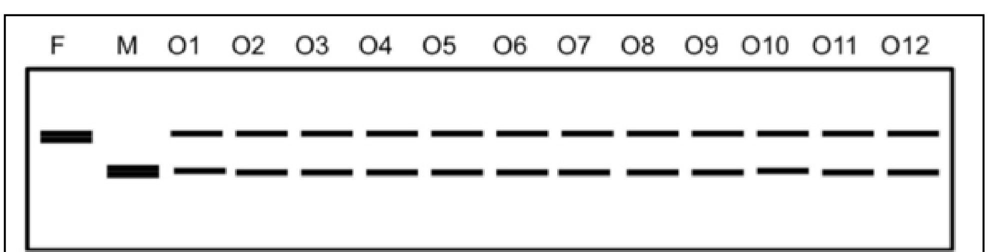

Figure 3. PCR printout example

fertilization of mom's egg with dad's sperm. *Phase 3* takes advantage of this knowledge and the understandings generated in *Phase 1* and *Phase 2*, to explore the relationships between genes, alleles, sperm and eggs. Staying at the genotype level, students use manipulatives to model allele behaviors and use mathematical strategies to quantify the relationships between parental contributions and offspring possibilities. The manipulatives enable students to model inheritance for up to two genes, and include eggs, sperm, alleles, male and female geckos. And the simulations performed with the manipulatives are algorithmically grounded, "imitating the processes that change content as well as the content itself" (Moulton & Kosslyn, p. 1276), as opposed to Punnett squares that are simulative but not emulative.

Although the module deals with both permutation and combination, and permutation is sufficient to describe allele assortment for an individual parent, there is more information to be derived from combinations involving both parents in a cross. Students are jigsawed in assignment to groups in the classroom, with each group examining one of the homozygous or heterozygous parental crosses for a one-gene system. Student groups then analyze the data across the different crosses, suggesting equations that might determine the total number of possible allele combinations C given a particular set of parental genotypes:

- maybe C = # egg types × # sperm type (1)
- maybe C = # egg types + # sperm type (2)
- maybe C = # types of alleles in female × # types of allele in male (3)

For an equation to be valid it must hold in all cases. While equations 1 and 3 work across all the cases, students see that equation 2 does not. They are able to see how the process for the allele mechanism is not an additive process because they are dealing with combinations. Using tree diagrams students are able to demonstrate convincingly for themselves that this is the case.

As they test their mathematics models on larger gene systems, they also find out the equation 3 does not work and they refine their biological thinking about what is happening in the system to make offspring. It is not about alleles combining; it is about egg and sperm combining. Having the biological justification tied to the mathematical model supports more consistent understanding of the process of inheritance.

It was stated earlier that rarity could be described as a probability using combination rules. If we think of probability, P, as a ratio, then P = desired outcome/all possible outcomes. Students can use equation 1, which is tightly connected to the inheritance rules students derived, as a way to find the denominator for P.

So now P = desired outcome/(*# egg types × # sperm types*) for each gene. The numerator is calculated in a similar manner to determine the frequency at which a

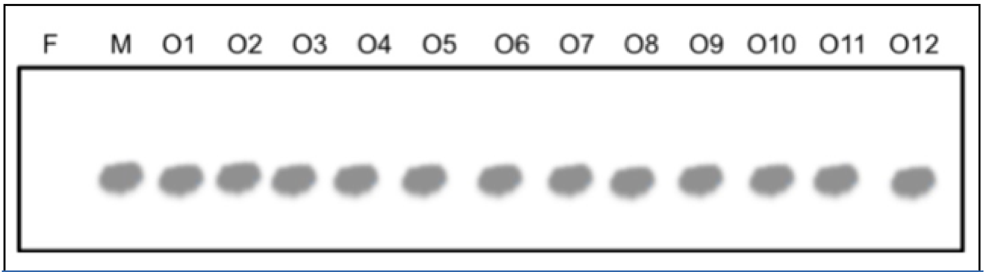

Figure 4. Western blot printout example

particular genotype can be expected from the mating of parents (for which the genotypes are known), although this expectation need not be consistent with observation of actual ratios.

Phase 4: Rules of expression. At this point in the unit, students recognize that they are still unable to solve their design challenge. There is still knowledge they have not reconciled for themselves at this point. While they are able to determine the number of possible genotypes an offspring might have and their theoretical ratios in a litter, they cannot predict what the gecko offspring will actually look like. This "wondering" guides the work students do in *Phase 4* to connect the genotype knowledge back to the phenotype of the organism, to understand how allele behaviors affect expression in the offspring.

Throughout the unit, students bring up dominance and recessiveness from their prior learning. However, they do not yet have any evidence to support their thinking due to our curricular choice to focus work at the genotype level and because allele behavior alone does not help students determine the offspring's appearance. During *Phase 4* we continue to analyze data at the genotype level by introducing another tool, the western blot, which (similarly to PCR) provides information about the allele at the even finer grain of RNA through protein variants (example shown in Figure 4).

This analysis provides students with evidence that requires them to think more carefully about dominance. Students are then confronted with another contrast case that pushes their thinking further. The key question repeated from *Phase 1*, "How can two sets of parents that look the same as each other produce offspring that look different?" drives this exploration. Looking at the crosses from three species of flowers (sweet peas, camellias, and snap dragons) for which the genetic data (PCR, western blot, and offspring genotype) all appear identical, one notices that the offspring for each cross looks different. Grappling with these pictures (which involve simple dominance, co-dominance and incomplete dominance) and the genetic information helps students reconcile both the relationships between genotype and phenotype and the concept of dominance with protein interactions. At this point, during the final design challenge, students take a summative pass at their presentation for the zoo.

Summary of Module's Critical Elements. There are three main ideas that we have maintained throughout the development and implementation of the module's instructional materials. All of these contribute in turn to a student's foundational confrontation with, and increasingly sophisticated grasp of, the biological processes of inheritance.

First, there is the genotypic data from cross-breeding that makes the patterns of inheritance more clear than inference from phenotypic data alone would allow. Neither data set alone is sufficient. Understanding inheritance necessitates access to both kinds of data and the ability to go back and forth between them until the connections become apparent.

Then there are the mathematical models that make possible the iterative discussion and presentation of inheritance concepts, specifically the connections between genotypic and phenotypic data. Without the artifact of a mathematical expression on which to base a design conversation about whether or not inheritance can even be described in such a manner (and if so, then how), no sophistication of understanding can take place.

Finally, there is the vehicle that engages student attention, displays immediate relevance to student level of ability, engenders increasing confidence in relation to the sophistication of the models and representations being created, and provides satisfaction in being able to bring inheritance processes to bear on a project goal: the application of the design challenge to breeding (Keller, 1987). For the students, biology moves from abstract process to concrete payoff: there is a genuine reason to learn this material, it helps them achieve some end. And it neither denigrates nor otherwise distracts from the purpose of learning about life sciences.

Learning Results: Qualitative Effects of Context Change on Student Participants

In this section we rely on a compilation of data, especially with regard to what we observed firsthand about students in the classroom, as well as what teachers told us about how students were engaging with the materials. We present three case studies of what transformation change in students looks like, as they move from difficult struggle to perceived success in this kind of curriculum context. The point is to give a qualitative feel for what positive effects might look like, to illustrate the ways in which some students struggled and then transformed their stance towards biology and the role played by mathematical understandings of biological phenomena (a more quantitatively oriented discussion is pursued in the next section).

In the first week of implementation some students were frustrated to the point of "shutting down" because answers to their questions were not automatically forthcoming, unlike the usual manner of instruction to which they had become accustomed. Other students were intrigued by this sudden change, and still others were entirely unreadable because they feared to give opinions and be considered "wrong" as a consequence. As we present our insights, we will also distinguish among them and demonstrate their differences.

Our first example concerns Alice (not her real name; no students' actual names are used in this paper in order to protect their confidentiality). Initially one of the unreadable students, she did not speak up until the third task, and even then limited herself to simple declarative observations and conjectures about the polymerase chain reaction results, "Female One had two thin bars and Female Two had one thick bar on the bottom…maybe the thick and thin bar cancel each other out, like dominant and recessive…the thin bars are recessive."

While student groups were displaying their first charts regarding one-gene simulation and debating how to express the outcomes mathematically, Alice's anxiety was visibly elevated, not only in the rising pitch of her voice and the stamping of her feet, but also in her attempts to rectify what she felt were inadequacies in her group's display. She did this by moving among the other groups and acquiring bits and pieces from their charts that she copied (inappropriately) onto her group's chart. "That sounds better than our answer," she said as she added a Punnett square, looking for white-out in order to obscure the correct work her group had done originally. She then appended allele combinations (that were not possible, but had appeared on someone else's list) to her group's list, culminating in the modification of her equation "male $\times$ female $=$ offspring" to be "1/2 male $\times$ 1/2 female $=$ offspring," because that is how someone else had written it. "This [poster] is so messy….yeah, that's the right answer."

Compare that with her interruption of the teacher after a two-gene simulation, when Alice produced the mathematical expression that the class was looking for, announcing, "It would be types of combination of egg times types of combination of sperm equals sixteen over here [pointing to a classmate's chart]. I can explain it. It works with one gene, too." Her teacher remarked in a post-class interview that without using the BLOOM module, Alice "wouldn't have come up with what she did and it will open up a lot of people's eyes" (i.e., raising her status in the classroom and increasing her self-confidence, which was becoming apparent from her now daily participation in class discussions). Furthermore, when faced with another round of charts from other groups, Alice made the decision not to copy another group's work "because they have different ones than ours," having developed an ability to distinguish among responses.

Note that Alice was not entirely reliant on the Punnett square, and that she bypassed the traditional pons asinorum of the expansion to two genes. Furthermore, her expansion led her to generalize the expression.

Our second example student, Ben, was worried at the beginning of the module. When the teacher commented to him that he looked stressed, Ben responded, "I don't get it. How do you mate these things?" before moving on to other work out of frustration. At that time it seemed to him that female traits were somehow "more dominant" than male traits; even after the one-gene simulation, he was still conflating alleles with different gecko species, perhaps due to exposure to so much terminology in a short span.

With manipulable material in hand, though, Ben was altogether different. Having finished the two-gene simulation, he was not merely able to discuss the relationship of alleles, genes, and gametes, he was enthusiastic about it. "I got this," he claimed as he listed the four permutations of gametes for the parent that the teacher had posed to the class. When asked to follow up with the number of offspring combinations, he made a diagram connecting each of the four permutations for one parent with one each of the four for the other, for a total of four and apparently missing the point, until a classmate questioned why each male gamete could not go with every female gamete? Ben then replied, "Are you talking about total number of combinations or offsprings... because you can have sixteen possibilities but only one sperm with one egg," showing a command of the subject matter that had been lacking only days before.

Indeed, shortly after Alice's derivation of the mathematical expression for the two-gene system, the teacher asked Ben to explain the expression to some other students who had been absent, which he then carried off with aplomb. In fact, Ben's change in context was so dramatic that he intended to carry it further once the module had ended, asking the teacher where he could acquire a gecko. He wanted to set up his own breeding concern.

Both Ben and Alice's epiphanies apparently arrived as the result of having multiple representations in front of them (in which the Punnett square was only one representation, and a minor one, at that). Note the key difference, however, being that Alice's is at least in part attributable to her analysis of the materials synthesized by her classmates (a design conversation played out as an iterative group effort) while Ben's relied more on brute forcing a breakthrough that required the collocation of the manipulables and both the genotypic and phenotypic data.

At the same time that the teacher noticed Ben "coming around," in a post-class interview she also commented on Cora, our third example student, "In the beginning they were shutting down…and Cora, that's her nature. She likes things the way they've always been." Indeed, Cora had not participated much during the first part of the module, and at the start of class one day she started to weep to such an extent that the teacher was obliged to help her from the classroom. "She really struggles with things that are uncertain and not secure," was the teacher's comment.

Cora kept trying, though, and when the class was reviewing the charts they were preparing to post about two-gene system combinations, Cora spoke up in her group. When another student asked about how many combinations they should get, she set him straight on how many he had found versus how many she had found. Later, she worked through a Punnett square that she had modified from a four-by-four grid in response to there being fewer combinations from homozygous parents. The discovery that she could change the square led her to interrogate how it could be made less complicated in order to serve a three-gene system.

First it occurred to her that a three-gene Punnett square was going to have $4 \times 4 \times 4 = 64$ boxes to fill in, and that was something her group had not considered. When another student asked if they had made an accounting mistake, Cora was ready to take charge. "Yes, I'll work on that," was soon followed by, "There! I fixed everything," an entirely reversed role from the student who had been led in tears from the class only a week or so before.

Again, Cora's revelation shared very little with either Ben's or Alice's. In this case, she did rely on the Punnett square more than the other two, but only to the point where the nature of its expansion (and her case, contraction) emerged from its associated bookkeeping. Once she could predict the range of the results, what had seemed wickedly ambiguous before became an exercise in precision, with which she was more than able to cope.

Learning Results: Quantitative Effects of Conceptual Change for Student Participants

The work of Schuchardt and Schunn (in press) provides a quantitative assessment of successive BLOOM module implementations with regard to student conceptual understanding and problem solving skills, and we will turn to that briefly for this section. Specifically, we want to demonstrate that "…changing the use of mathematics from (teacher-presented) calculated procedures to (student-developed) modeled processes that embed

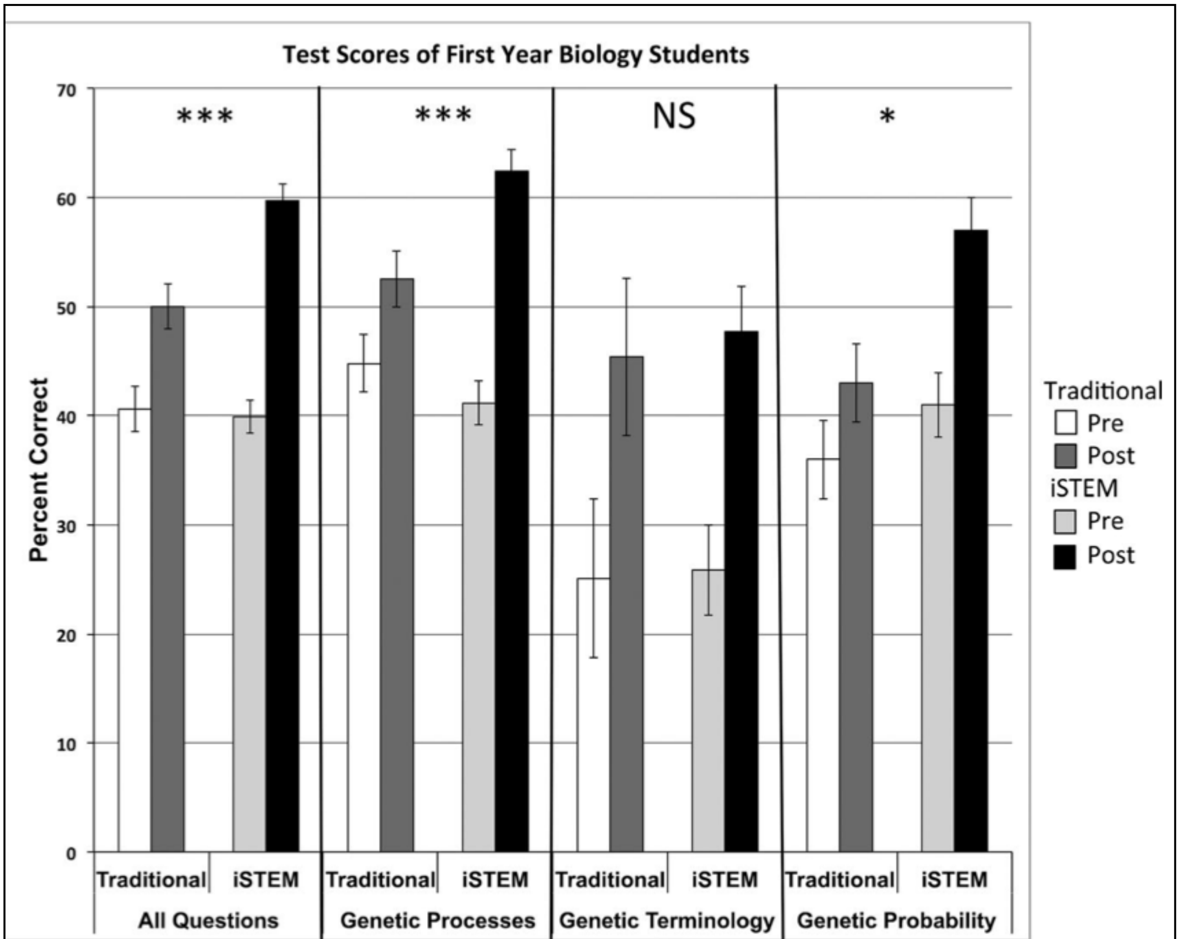

Figure 5. Comparative test scores for control group versus iSTEM (integrated STEM being the description of the BLOOM module) treatment group, NS >.1, *p <.05, ***p <.001.

biological concepts within the mathematics…" does have measurable effects, being complementary to the examples reported for this study. It should be noted that Schuchardt and Schunn's sample size far exceeds that for this study, giving generalizable inferences a statistical basis.

In Figure 5 Schuchardt and Schunn compare the results of student testing on control and treatment groups with respect to the topics of genetic terminology, processes, and probability in inheritance (and in general across all three topics), the same as we are dealing with in this study. With the exception of terminology, for which scores were similar, the significant differences favoring beneficial results are readily apparent.

Teaching Results: Teachers and Their Implementations of a BLOOM Module

Although we treat the sense-making required of our teachers more in depth in another forum (Cox, Reynolds, Schuchardt, & Schunn, in press), it is appropriate to include a glimpse of those insights here, as well. We present abbreviated accounts of two very different teachers with respect to their attitudes toward the module, and their approaches to implementation. It is worth noting that all the participants were similar in remembering their own secondary educational experience with biology as being devoid of mathematics.

Deborah participated in several implementations of the module. Her familiarity with the changes that have accompanied the iterations has established a level of trust in the designer's intent and the materials' utility. Approaching retirement, she also has the most experience in the classroom of all the participants. For her, mathematics is a medium necessary for analysis and presentation of data, and there clearly is not enough of it in general biology today. She is dedicated to making that happen to the extent of experimenting with innovative materials, or, as she says, "A teacher has to be open to seeing differently or kids won't look at [content] another way."

She believes that more biochemistry and its accompanying mathematics is needed in the biology curriculum where she teaches. Likewise, any preparation for physiology studies must include mathematics because "everything for physiology has an equation."

Edna, on the other hand, remains only tentatively in favor of the BLOOM module's mathematical emphasis. She is unhappy with the ambiguity of topics that eluded resolution, as in whether there were three or four different products of a monohybrid cross (genotypic *AA, Aa, aa* versus algebraic *AA, Aa, aA, AA*); she feels it produces unnecessary anxiety in the students when she makes them answer their own questions. For her, teaching biology is challenge enough without bringing mathematics into the picture:

> *In the science classroom, having to teach so much content in a short amount of time, I think trying to find a way to incorporate the math – on my own – is just a bigger challenge for me at this point. I've only been teaching for five years, so maybe I'm still trying to learn how to teach the science content?...This is probably my first year that I actually feel comfortable that I don't have to keep developing things. Y'know, the things that I've already developed have been working. I'm just kind of tweaking things here and there. I might be able to incorporate math here or there in something in my tweaking for future use, but to start from scratch and develop a whole unit or a whole lesson that does incorporate the math? I'd probably say, no, I wouldn't.*

One recurrent theme that all the participants focused on was their perception of emergent self-efficacy among the students. For Deborah, who embraced the BLOOM materials, this effect was accelerated by the module, but not entirely attributable to it. That is, she felt self-efficacy would appear anyway, albeit over a longer time. Edna was progressively convinced over a series of implementations that there was a change in kind rather than degree, that self-efficacy now occurred where none had before, but this was still not enough to convince her of her own self-efficacy with regard to introducing mathematics on her own.

Conclusions

As a professor of biology at Cambridge University, Reginald Punnett was in at the first stages of recognizing inheritance as a scientific process. Since early in the Twentieth Century, his eponymous square has provided an innovative device for keeping track of very simple allele interactions, making that concept accessible to a wide audience of university academics. In the meantime, however, secondary level students of introductory biology have obtained calculational resources that supersede the Punnett square's usefulness for anything beyond the most elementary applications (not to mention obviating the wide spread confusion associated with merely expanding the square from one allele to two, as discussed in Moll & Allen, 1987, and Tolman, 1982).

Revisiting the disputable utility of the Punnett square as an example of where introducing mathematics might promote secondary level students' understanding of biological processes, we have presented here:

- an argument for the benefits from mathematics emphases and engineering design to students of biology
- the typical structure of an instructional module with which we pursued these benefits, and a detailed example of such a module
- some qualitative insights from the implementations we have made to date.

Because we find the general response from students and teachers to be positive, we intend to continue in the rapid-prototype development and implementation of this module, and similar ones on various biology topics, with the intent of purposefully disseminating the associated materials and methods.

In this particular module, mathematics augmented student understanding of the exponential complexity of allele combinations and, with students' repeated effort, gave those students a way to deal with any number of alleles that Punnett squares and even manipulatives do not provide in a practical sense. Because the elements of the mathematical expression are strongly tied to the actual biological processes of inheritance there is an affordance for reinforcement of understanding these processes through multiple pictorial and graphic representations related to quantification.

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