Pre-Service Teachers' Pedagogical Content Knowledge In Transferring From Basic Musical Notations To Basic Fractions

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Abstract

We report on mixed methods research that explored pre-service teachers' pedagogical content knowledge in basic musical methods for teaching and learning of basic fractions within the framework of Vygotsky's Theory of Scaffolding. Vygotsky's use of scaffolding opines that knowledge, skills and prior experiences, create the foundation for potential development. This allows students to interact with adults (teachers) and/or peers in STEM domains to accomplish tasks, which could possibly not be completed independently. In accomplishing these tasks, the quasi-experimental pre-post test design was used to explore the preservice teachers' pedagogical content knowledge in music notations and fractions in non-equivalent control group design. The findings showed evidence of preservice teachers successfully transitioning from basic musical methods to introductory fractions in providing realistic mathematics discourse in the classroom. As a result, we recommend approaches that build these synergies for efficient and effective classroom mathematics discourse.

Keywords: basic musical methods, basic fractions, pedagogical content knowledge, preservice teachers, Vygotsky's Theory of Scaffolding

Introduction

Research by Schmidt-Jones(2008), Shah(2010), An and Tillman(2014), An, Tillman, Boren, Wang(2014), and Farnham (2015) have established numerous pedagogical links between mathematical relations, proportions, integers, logarithms, arithmetical operations, trigonometry and geometry to music melody, rhythm, intervals, scales, harmony, and tuning. These naturally occurring overlaps between music and mathematics and by far other STEM courses, offer opportunities for preservice teachers to create social engagements and interactions with music-themed pedagogical content knowledge and activities in mathematics. For instance, Vygotsky promoted the development of higher level thinking and problem solving strategies that utilize critical thinking skills, thought processes and such scientific processes of observation, experimentation and inferences help pupils to gain new knowledge, and the knowledge achieved also serves as a foundation for the behaviours of every pupil (Future Learn, 2020). Narrowing the knowledge down to mathematics, one observes that the socio-cultural interactions in the Ghanaian primary classroom embody a lot of music and fraction activities that should equip preservice teachers to acquire pedagogical content knowledge to teach any STEM subject. They then subsequently and gradually transfer the knowledge to their pupils through scaffolded interventions. The three main basic music activities readily available for Vygotsky's scaffolding are pitchnotes, time durations and style dynamics (Farnham, 2015).

Vygotsky's theory of scaffolding states that learners learn first from person-to-person interactions and then individually through an internalization process that leads to deep understanding without the scaffolds, where social interactions create conditions to participate and extend knowledge to the highest level (Vygotsky, 1978; Matusov, 2015). Scaffolding emerges from instructional conversations, cognitive apprenticeship, practice-and-problem-based learning, dialogic inquiry, and pedagogy and didactics that share common principles, practices and pedagogies with sociocultural interactions to address issues of mastery, motivation, problem-solving strategies and didactic situations as scaffolds are gradually removed (Blake & Pope, 2008; Turuk, 2008). However, when scaffolding is misunderstood understood as direct instruction, the applications may be limited to STEM education. Scaffolding was broadly conceived to comprehensively encompass such wider STEM techniques of proving adult support, demonstrating the sequences, dividing tasks into simpler steps, providing guidelines, keeping attention and breaking content into manageable pieces (Verenikina, 2003).

Musical notes, frequencies, and sound waves reveal many opportunities to leverage music to support the learning of STEM-related subject matter in basic schools all over the world. The basic sound synthesizer also provides a cornucopia of fun material related to higher-level science principles. The opportunities for fun learning are nearly limitless. Making STEM education a fun process for pupils of all ages, levels or classes help to provide highly-trained engineers, scientists, and mathematicians to propel the economies of all nations (Scientific Search, 2014). In music, studies (Salisbury, 2011; Edwards, 2012) have revealed that notes, whole is Semibreve, one-half is Minim, one-quarter is Crotchet, one-eighth is Quaver, one-sixteenth is Semiquaver, and one-thirty-second is Demisemiquaver. In time durations, whole rest is whole, half rest is one-half, quarter rest is one-quarter, eighth rest is one-eighth, sixteenth rest is one-sixteenth and thirty-second rest is one-thirty-second. By extension, a dotted whole is whole and one-half, a dotted one-half is one-half and one-quarter, a dotted one-quarter is one-quarter and one-eighth, a dotted eighth is one-eighth and one-sixteenth, a dotted sixteenth is one-sixteenth and one-thirty-second, and a doted thirty-second is thirty-second and one-sixty-fourth (An, & Tillman, 2014)). In style dynamics markings, ***mf*** as moderately loud is whole, ***f*** as loud is one-half, ***ff*** as very loud is one-quarter, and ***fff*** as very very loud is one-eighth, or alternatively, ***mp*** as moderately soft is whole, ***p*** ass oft is one-half, ***pp*** as very soft is one-quarter, and ***ppp*** as very very soft is one-eighth. That is, by participating in these scaffolds, preservice teachers and their pupils engage in social interactions (Fan, 2010).

Despite these music endowments in fractions, preservice teachers lack pedagogical content knowledge to integrate formal music notations to social interactions during routine singing and dancing activities into the teaching and learning of fractions (McGuire, 2004; Blake, & Pope, 2008).This may be partly attributed to their lack of deep conceptual knowledge of fractions and frequently misrepresentations of the meanings, names and notations in mathematics discourses. Again, preservice teachers fail to properly conceive the abstract representations in physical, social and cultural constructs with scaffolded activities in music in order to gradually and systematically transit into the mathematics concepts meaningfully, practically and real life-centred (An, Capraro, & Tillman, 2013).

Of all the areas of STEM and in particular mathematics, fractions were deemed most appropriate. This is because fractions are foundational for mathematics study

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beyond primary school and competency in fractions is a strong predictor for high school (and beyond) mathematics performance. In addition, fractions have practical applications to all STEM domains and everyday live. In fact, fractions are strongly related to musical rhythms through clapping, singing and dancing that do not require specialized instruction and can be accessible to all levels of musical training. This does not impose specialized musical training on the mathematics teachers (Hamilton, Doai, Milne, & Saisanas, 2018). This is particularly useful for the complex and multifaceted concepts in fractions that require diverse and integrated fields of study. Cross disciplinary and interdisciplinary nature of music notations and STEM education is very essential in depicting the true meaning and physical manifestations of fractions (Wright, 2009).Studies (An, Ma, & Capraro, 2011) have actually established that pedagogical content knowledge of preservice teachers in the music-oriented social interactions empower preservice teachers to achieve conceptual mastery, deep understanding and competencies infraction conceptualizations. However, this seems absent in our part of the world. Preservice teachers, who came to the university to improve upon their professional knowledge and skills, differ on a number of variables. Prominent among these are gender, class level and programmes of study (Cimpian et al., 2016). Based on these differences, the researchers came out with the following research questions that could adequately to address the yawning gaps in teaching and learning:

1. Are there differences in gender, class level and programmes of preservice teachers in transferring basic musical notations to fractions?
2. In using the basic musical notes, how do preservice teachers' enjoy the transference from basic musical notations to basic fractions?

Methodology

The research design and methods

The researchers used the quasi-experimental prepost test design was used to explore the preservice teachers' pedagogical content knowledge in music notations and fractions in non-equivalent control group design. The researchers first administered the pretest to the preservice teachers to assess their mathematics dispositions before the intervention in order to explore the two groups' initial differences, avoid selection bias and examine changes in pedagogical content knowledge. In doing this experiment, the experimental stage second was randomly assigned to the lecturer utilizing music-mathematics integrated interventions and the conventional first stage lesson was assigned to the lecturer using only standard mathematics lectures. This first really helped in controlling for most threats to internal validity, particularly history, maturation, testing, instrumentation and regression. The random assignment of the participants helped to neutralize any differences between the two groups, reduce potential causes of confounding variables and assess the impacts of the music-fraction intervention. Participant mortality was preserved throughout the entire study since every preservice teacher who began the study finished it.

Sample and sampling technique

The study was conducted in the Department of Basic Education in the University of Education, Winneba, where the participants who taught across from the kindergarten, primary and junior high schools were grouped heterogeneously across gender, class levels of teaching and programmes of teaching. There were 34 preservice teachers selected for the study. The Department was selected on two criteria---it has got full-time Senior Lecturers who teach comprehensive and sequential music education programme based on the University's standards of Music Education for the basic levels of education and ensure that construct and content validities were checked, and preservice teachers who had taught mathematics verified the standard protocols and principles of teaching fractions in basic schools without musical notations to ensure that neither group had already been exposed to music and fractions. To test the internal consistency of test instruments, the tests were analyzed via the Kuder-Richardson 20 formula to obtain reliability coefficients of 0.70 for the experimental group, and 0.72 0 for the control group (Creswell, 2014).

Instruments of data collection

The teachers used teacher-made test instrument to collect the data. In the procedure for collecting the data, and before the experimental group received the interdisciplinary instructional interventions, the researchers administered a researcher-designed pretest on fractions to both groups in order to provide baseline data and outline adequate guidelines on the nature of questions and completed pretest answers during their regularly scheduled periods. Following the pretest, the experimental group received four 60 minute interventions during the regular scheduled periods involving oral, aural, and visual interactions, representations and discourses between music and fractions. The control group received alternate lessons at the same times without the transference of music notations.

The intervention procedures started with scaffolding music notations and gradually removed the scaffolds as pupils transit into more challenging and complex tasks and activities in fractions based on Vygotsky's theory. The social interactions and activities pupils performed to match the music notations were mainly clapping, walking, jumping, running, singing and babbling that resonate well with the respective notes, rests, dynamics and their dotted forms. These cross disciplinary social interactions brought to the fore the essential features of Vygotsky's social milieu between music and mathematics pedagogies, music and mathematics content, and music and mathematics knowledge in the aural and kinaesthetic activities. Then, after these activities, the researchers administered the posttest both groups (An, Capraro, &, Tillman, 2013).

Instruments of data analysis

The researchers used the independent sample t-test, analysis of variance (ANOVA) and 5-point attitudinal scale to analyse the data. The independent sample t-tests met the requirements of continuous dependent variables, categorical independents, cases having values on both the dependent and independent variables, independent observations of samples/groups, no relationship between the subjects, random samples of data from the population, normal distribution of the dependent variables for each group, homogeneity of variances and no outliers. The t-test was used to test the statistical differences between the means of two groups, statistical differences between the means of two interventions and statistical differences between the means of two change scores. Because the independent samples t-test could only compare the means for two (male and female) groups, the researchers used the ANOVA to make comparisons among groups in the class levels and programmes of the preservice teachers (Creswell, 2014). The Levene's test (H_0: the population variances of group 1 and 2 are equal) was very crucial in making a judgment about the null hypothesis (H_0). Thus, we would reject the null hypothesis if the two groups are not equal. If Levene's test indicates that the variances are equal across the two groups (i.e., *p*-value large), we would use *equal variances assumed*, and if otherwise (i.e., *p*-value small) we would use *equal variances not assumed*. In the t-test for equality of means, we used the computed test statistic, *Sig (2-tailed) and Mean Difference* to interpret the results. Particularly, positive mean difference meant male group performed significantly greater than the mean for the female group, and smaller p-values less than .05 meant that we rejected the null hypothesis (Creswell, 2014).

Furthermore, the ANOVA was used to test the degree of differences between more groups in levels and prgrammes in the experiments. If the obtained p-value from the ANOVA table was less than or equivalent to the level of significance, we rejected the null hypothesis and concluded that all the population's means were not equal. If the obtained p-value from the ANOVA table was greater than the level of significance, we failed to reject the null hypothesis and concluded that all the population means were equal (Creswell, 2014). Alternatively, we rejected the null hypothesis whenever the F value was much larger than the critical value, and concluded that there was a statistically significant difference among the population means (Creswell, 2014).

Lastly, the attitudinal scale was used to support and explain the results of the t-test and ANOVA test. The Attitude scale provided a quantitative measurement of attitudes, opinions or values by summarising numerical scores given by researchers to the preservice teachers' responses. Even though the different types of scale, namely the presentation of a series of statements, response to 'strongly agree; 'agree'; 'undecided'; 'disagree'; or 'strongly disagree', and the scoring of responses into a numerical value (e.g. 1 to 5 on each statement), the researchers opted for the latter. The latter summarises statements and transforms them into graphical form easier to read and interpret (Payne, & Payne, 2011).

This study did not only add knowledge to reinforce the active manifestations of Vygotsky's theory to scaffolding but also brought to strong focus the techniques of using music to teach mathematics. In particular, the experimental designs were fully exploited within STEM domains, and the principles of drawing STEM domains closer to social interactions tremendous improved in upon teaching and learning of mathematics (fractions).

Results

Based on the two research questions, the results of Tables 1 and 2 and Figure 3 have been displayed on the t-test, ANOVA test and attitudinal scales of the preservice teachers. Research question contains the t-test was used for Table 1 and ANOVA test for Table 2. Research question 2 contains the attitudinal scale for Figure 1. Based on research question one, the following tables seeks to identify the statistical differences between the gender groups. The reason is to establish whether gender could be a hindrance in transferring musical notations to fractions.

In Table 1, the Levene's test (H_0: the population variances of male and female are equal) shows that the variances are equal across the two groups (i.e., *p*-value large). Therefore, we would use *equal variances assumed* to make judgment of our hypothesis and conclude that the population variances were equal for male and female groups. In the t-test for equality of means, there were positive mean differences in rests (proper fractions) and styles (improper fractions). This means that male group performed significantly greater than the female group in only rests and styles. However, in notes (both proper and mixed fractions) and style (proper fractions), there were negative mean differences, and this shows that males did not perform significantly greater than females. Therefore, male and female preservice teachers interacted well in most of the cases. Hence, social interactions as concepts in Vygotsky's theory did not only marry music and fractions but brought diverse ideas emanating from both gender to influence the knowledge of music on mathematics discourses.

Since Table 1 gave us this split decision on the statistical significances, we performed the ANOVA test in Table 2 to further explore the impact of music on the levels of teaching mathematics in school, as measured by the scores.

		Levene's Test		T-Test for Equality of Means					95% Confidence	
Interactions		F	Sig.	t	df	Sig.	Mean Difference	Std. Error Difference	Lower	Upper
Notes with proper fractions	Equal variances assumed	0.178	0.676	-0.548	33	0.588	-0.124	0.227	-0.586	0.337
	Equal not assumed			-0.548	32.945	0.587	-0.124	0.227	-0.585	0.337
Notes with mixed fractions	Equal variances assumed	0.176	0.678	-0.282	33	0.780	-0.046	0.162	-0.376	0.284
	Equal not assumed			-0.281	30.818	0.781	-0.046	0.163	-0.378	0.287
Rests with proper fractions	Equal variances assumed	0.774	0.385	0.952	33	0.348	0.193	0.203	-0.219	0.605
	Equal not assumed			0.949	32.352	0.350	0.193	0.203	-0.221	0.606
Rests with mixed fractions	Equal variances assumed	0.648	0.427	-0.971	33	0.339	-0.111	0.114	0-.344	0.122
	Equal not assumed			-0.968	32.296	0.340	-0.111	0.115	-0.345	0.122
Styles with proper fractions	Equal variances assumed	0.396	0.533	-1.071	33	0.292	-0.203	0.189	-0.588	0.182
	Equal not assumed			-1.066	31.385	0.295	-0.203	0.190	-0.590	0.185
Styles with mixed fractions	Equal variances assumed	0.614	0.439	0.573	33	0.570	0.131	0.228	-0.333	0.595
	Equal not assumed			0.572	32.363	0.571	0.131	0.229	-0.335	0.596

Table 1. Gender differences in musical notes, rests and styles in fractions

Interactions	Groups	Sum of Squares	df	Mean Square	F	Sig.
Notes with proper fractions	Between Groups	1.025	2	0.512	0.365	0.697
	Within Groups	44.861	32	1.402		
	Total	45.886	34			
Notes with Mixed fractions	Between Groups	2.805	2	1.403	0.416	0.663
	Within Groups	107.938	32	3.373		
	Total	110.743	34			
Rests with proper fractions	Between Groups	2.821	2	1.411	0.419	0.661
	Within Groups	107.750	32	3.367		
	Total	110.571	34			
Rests with mixed fractions	Between Groups	11.632	2	5.816	2.617	0.089
	Within Groups	71.111	32	2.222		
	Total	82.743	34			
Dynamics with proper fractions	Between Groups	17.470	2	8.735	3.280	0.051
	Within Groups	85.215	32	2.663		
	Total	102.686	34			
Dynamics with mixed fractions	Between Groups	4.341	2	2.171	0.643	0.532
	Within Groups	107.944	32	3.373		
	Total	112.286	34			

Table 2. Class level differences in musical notes, rests and styles in fractions

In Table 2, the class levels of the preservice teachers were divided into kindergarten, primary and junior high school. We earlier postulated that if the obtained p-value was greater than the level of significance of .05, we would fail to reject the null hypothesis and concluded that all the population means were equal (Creswell, 2014). And we would reject the null hypothesis whenever the F-value was much larger than the critical value, and concluded that there was a statistically significant difference among the population means (Creswell, 2014). It was evidently clear that there were statistically significant differences [p>0.05]. Despite reaching no statistical significance, the actual differences in mean scores between the groups were quite small with effect size (eta squared = 0.01). Once again, there were enough cross interactions between and among the various levels of teachers in exploring music to teach and learn fractions. It was also discovered that the social interactions were much more experienced during music notes as compared to the music rests and dynamics. This confirms our earlier notion that Ghanaian schools utilize rhythms during classroom plays, singing and dancing than the more advanced rests and dynamics.

Table 3 contains another one-way between-group ANOVA conducted to explore the impact of the interactions on subjects preservice teachers teach, as measured by the scores. The subjects were divided into four groups (Group 1: science/mathematics; Group 2: social/religious

Interactions	Groups	Sum of Squares	df	Mean Square	F	Sig.
Notes with proper fractions	Between Groups	1.784	3	0.595	1.398	0.262
	Within Groups	13.188	31	0.425		
	Total	14.971	34			
Notes with Mixed fractions	Between Groups	0.933	3	0.311	1.447	0.248
	Within Groups	6.667	31	0.215		
	Total	7.600	34			
Rests with proper fractions	Between Groups	1.067	3	0.356	0.993	0.409
	Within Groups	11.104	31	0.358		
	Total	12.171	34			
Rests with mixed fractions	Between Groups	0.615	3	0.205	1.943	0.143
	Within Groups	3.271	31	0.106		
	Total	3.886	34			
Dynamics with proper fractions	Between Groups	2.332	3	0.777	2.884	0.051
	Within Groups	8.354	31	0.269		
	Total	10.686	34			
Dynamics with mixed fractions	Between Groups	2.372	3	0.791	1.919	0.147
	Within Groups	12.771	31	0.412		
	Total	15.143	34			

Table 3. Programme differences in musical notes, rests and styles in fractions

studies; Group 3: languages/culture and Group 4: pretechnical and vocational skills).

Once again, on Table 3, the p-value was greater than.05, and we would fail to reject the null hypothesis and concluded that all the population means were equa). Also, the F-value was much larger than the critical value, and concluded that there was a statistically significant difference among the population means (Creswell, 2014). Therefore, the results show that there were no statistically significant differences among the preservice teachers based on the subjects they were teaching in the basic schools [p>0.05]. Despite reaching no statistical significance, actual differences in mean scores between the groups were quite small with effect size (eta squared = 0.01). This means the social interactions between music and fractions influenced all subject preservice teachers and not only those who teach mathematics. In other words, preservice teachers of various professional backgrounds and practices require Vygotsky's social interaction theory to scaffold mathematics and other subjects in school to improve upon academic performance.

Based on research two, the following figure was extracted to measure the preservice teachers' particular and enjoyment of the basic musical notes, rests and styles. Responses denoting 'enjoyable', 'very enjoyable' and 'extremely enjoyable' denote good expectations. In such cases, it could be concluded that there was a significant impacts of the music on fractions.

Figure 1 highlights the various levels of interactions preservice teachers ranked themselves during the music-fraction discourses. It was observed that many preservice teachers generally enjoyed the interactions between the two subjects in the teaching and learning fractions. These interactions ultimately foster and promote pedagogical content knowledge of preservice teachers in fractions. Following the results of Table 2 and 3, it could be deduce that no differences in the characteristics helped the students to develop positive and effective attitudes for learning fractions using music.

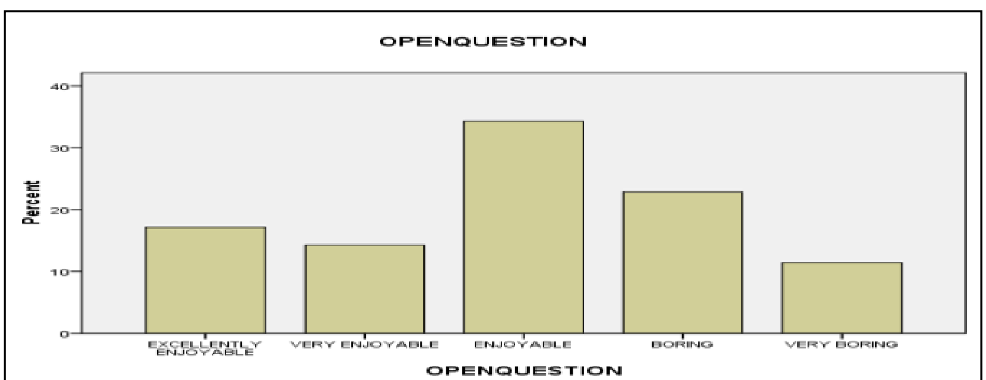

Figure 1. Preservice teachers' expectations of musical notes, rests and styles in fractions

Discussion of Findings

Given the ordinal nature of the data collected and the relatively small equal sample sizes employed, the gain scores of the groups were compared using independent t-test and ANOVA statistics. All the results show that the groups' gain scores ranked significantly higher even with no statistically significant differences in the gender, levels and programmes in both proper and mixed fraction discourses. The researchers therefore, deduced that like An, Capraro, and Tillman (2013), social interactions with scaffolds among the participants were the most important tools that promoted and fostered teaching and learning of fractions. The typical Ghanaian classroom is embroiled with music and dance and if properly harnessed into teaching and learning, could raise the standard of content knowledge of pupils and teachers.

Although the two research questions were supported positively by the data, they should be interpreted cautiously. This is because majority of the preservice teachers in the experimental group could write proper and mixed fractions using musical methods in some of the items but were not consistent across all the items of the posttest. For instance, no preservice teacher had a gain score on addition of mixed fractions using the interactions of the dotted notes, rests and dynamics. However, the researchers take consolation on studies (An, Ma, &Capraro, 2011; An, Capraro, &, Tillman, 2013) that contend that fraction beginners should not engage in complex calculations musical notations due to their lack of maturity and previous experiences. That notwithstanding, since neither group had been exposed to fractions containing music notations before this study, the successes of the preservice teachers was readily attributed to the social interactions of music-mathematics concord.

Conclusion And Implications

The hypothesis that integrated experiences in music and fraction activities helped the preservice teachers to represent and compute fractions. The implications of the findings were that using music and other liberal arts contextualize mathematics concepts (fractions) further boost mathematics skills. The enjoyable of sounds, notes and pitches do not only bring active learning but alos help transfer concepts from other disciplines to STEM education in the 21st century, especially whenVygotsky's Theory of Scaffolding is fully exploited (Verenikina, 2003). This explains why the tests generally showed statistically significant differences across gender, class levels and programmes of preservice teachers. These results mean that there were adequate social interactions among the preservice students. However, we suggest that researchers should employ rigorous qualitative techniques and analysis in order to isolate preservice teachers' verbalizations and rhythms regarding their thought processes when engaging in music-mathematics with traditional mathematical fraction symbols.

Secondly, the findings of preservice teachers' expectations showed the music activities raised substantially good tools for equipping their pedagogical content knowledge to handle fractions (Scientific Search, 2014). However, future research should explore longitudinal implications of achieving early successes with fractions using musical notations. Such a study could not address whether preservice teachers' knowledge in music in colleges of education had impacts on the test scores on fractions in the university. Future researchers should explore preservice teachers' pedagogical avenues earlier in order to foster understanding in the university..

In addition, the findings showed that education and training of teachers must aim at building cross discipline pedagogies. The cross disciplinary interactions between music and mathematics in this study clearly boosted the Vygotsky's theory of scaffolding. However, the findings should not be interpreted as lending support to unsubstantiated claims that instruction in other disciplines will inevitably produce positive effects. The significance of this quasi-experimental exploratory study lies in the possibility of preservice teachers using musical methods to provide quality and effective mathematics instructions.

Lastly, the findings postulated that when preservice teachers with various backgrounds and social experiences work together, some might become beneficiaries of the synergies as done in STEM education. It should not, therefore, be interpreted to mean that other methods of teaching fractions are inferior to the approach used here. Just as cooperative synergy with music is necessary in order to produce quality and effective teaching and learning of mathematics, mathematics should equally help the teaching and learning of music and incorporate the idea into the other STEM domains (Hamilton, Doai, Milne, & Saisanas, 2018). Preservice teachers who have multiple teaching and learning methodologies at their disposal may foster their pupils' learning outcomes from the various strategies and styles in the classroom.

Interested researchers might seek further evidence of preservice teachers' ability to grasp fraction concepts at each Vygotsky's developmental levels. Furthermore, it would be more thought provoking to determine how alternate forms of instruction affect music-mathematics synergies beyond notations and additions of basic fractions.

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