

A Phenomenological Study of Adinkra Values in Exemplifying Concreteness Fading in Multiplication of Numbers

Clement Ayarebilla Ali

University of Education, Winneba, Ghana

Abstract

This study sought to use Adinkra artefacts to present “Concreteness Fading” in the multiplication of one-digit and one-digit numbers. In a descriptive case study, the study described the phenomenon in detail in its real-world context. The researcher used a typical case purposive sampling technique to select 51 participants from 300 student teachers. The typical case sampling obtained a sample that was representative of the typical experiences to understand the most common experiences within the population. Two stages of data collection were employed. The first was the initial commenting of the pretest and the second was the actual codings of their responses. Utilizing the interpretive phenomenological analysis to give full appreciation to each participant’s case, the results were equally presented in two stages. This enabled the researcher to code each stage of the analysis of the emergent themes on the multiplication before moving to the superordinate themes on the values of Adinkra. Following the findings, we concluded that heavy use of only concrete objects and examples without abstracting can harm teaching mathematics. It was recommended that student teachers should always avoid rushing to symbolic manipulations of mathematics but rather strengthen their pedagogies.

Keywords: Adinkra values, concreteness fading, multiplication of numbers, phenomenological study, purposive sampling

Introduction

Concreteness Fading was originally proposed by Jerome Brunner and has three developmental stages, namely iconic and symbolic (Bruner, 1966; Kim, 2020). The concept was later modified to Concrete, Representational, Abstract (CRA), or Concreteness Fading to refer to a three-step Pearce & Orr, 2018). Through fading, student-teachers can “empty” the learned concepts of its specific sensory and perceptual properties, so they can grasp its formal, abstract properties (Pearce & Orr, 2018). Schalk, 2020; McNeil & Fyfe, 2012; Pearce & Orr, 2018; Pickering, 2023; Suh et al., 2020) suggests that concrete representations get “faded” to yield more generalisable both during and after instruction. The abstract nature of the indigenous artefacts and their applications to the teaching and

learning of mathematics require conscious fading.

Despite the long existence of “Concreteness Fading”, it remains much swallowed! Moreover, it has evolved with many variants, making it difficult to know where to fit indigenous artefacts (Suh et al., 2020). Kokkonen, Lichtenberger, and Schalk (2022) make more encouraging evidence for the effectiveness of concreteness fading. McNeil and Fyfe (2012) contend that no study has experimentally tested the effects of concrete-to-abstract representations. It is even inconceivable to embed artefacts like ‘Adinkra’ into “Concreteness Fading”. The current experimentation by Aduko and Armah (2022), and Donovan and Fyfe (2022) revealed significant differences between groups, suggesting potential benefits of concreteness fading. This study proffers local indigenous “Adinkra” artefacts that have always been glossed over. The virtues and values of these indigenous materials in mathematics learning cannot be over-sympathised (Ali, 2022).

An Overview of ‘Adinkra’ Artefacts

The Akan word ‘Adinkra’ means “Farewell” (Appiah & Nartey, 2015). ‘Adinkra’ are traditional symbols that are primarily a usual translation of thought and ideas, expressing and symbolizing the values and beliefs of the people among whom they occur (Ali, & Anderson, 2021; Ali, 2021). These symbols are embossed on textiles, pottery, stools, umbrella tops, linguist staff, logos, clothes, furniture, sculpture, earthenware pots, and many others (Babbitt, Lachney, Bulley & Eglash, 2015; Boddy-Evans, 2020; Kuwornu-Adjaottor, Appiah & Nartey, progression from a physical, diagram, and abstract states (Kokkonen & Schalk, 2020; Research 2015). Even though Adinkra symbols were not originally used to teach mathematics (Okyere 2021), they still communicate mathematics ideas, values, and knowledge. Therefore, the link of “Adinkra” symbols, and its immersed contributions to the teaching and learning of mathematics, in this context, multiplication of 1-digit by 2-digit numbers, was the major motivation.

Adinkra symbols have mathematics digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), operators (+, -, ×, ÷), and relaters (=, >, <, ≥, ≤) (Douglas et al., 2020). This brings to bear the context of mathematics in indigenous settings (Ali, & Anderson, 2021; Ali, 2021; Ali, Davis, & Agyei, 2021). Ali and Anderson found that musical symbols are math-

ematically oriented and can be used to teach and learn mathematics. Ali (2021) explained how symbols are very perfect in the teaching and learning of mathematics. Most importantly, Ali, Davis, and Agyei (2021) showed how local indigenous symbols contextualized mathematics and must always be explored to the fullest. There is therefore no saying that Adinkra which is prevalent in most cultures can be exploited for the achievement of mathematics. Instead of the above literature, how can “Adinkra” symbols help student teachers perform tasks in the three stages of multiplication of 1-digit and 2-digit numbers? The following research questions seek to address this question:

Research Questions

In answering this research problem, four research questions and their tasks were used with “Adinkra” symbols. These included “Friendship”, “Power of Love”, “Strength” and “Intelligence”.

1. What value does ‘friendship’ bring to the multiplication of numbers?
2. How does the value of ‘love’ help in the multiplication of numbers?
3. Why does the value of ‘strength’ help in the multiplication of numbers?
4. For whom does the value of ‘intelligence’ help in the multiplication of numbers?

Methods

Figure 1 shows a sample of nine ‘Adinkra’ symbols that the student teachers used to preview the concrete

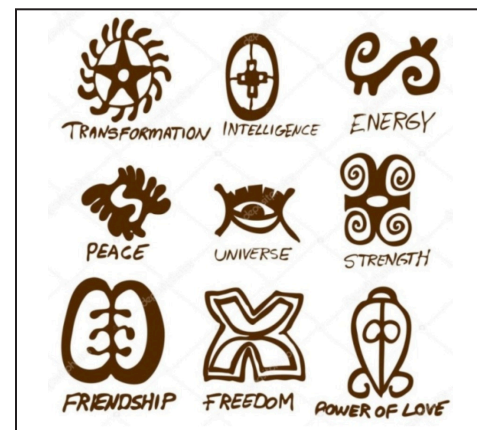


Figure 1. Indigenous Ghanaian “Adinkra” Symbols (Efiabevi, 2013)

representations. They embossed them with interesting and thought-provoking English language interpretations. Some of these meanings are 'friendship', 'power of love', and 'intelligence'. I selected the nine symbols purposely to use their mathematical values to motivate the student teachers to take up such positive values and actively participate in the experiments

The Design

The researcher chose the case study design. Jross (2023) defines a case study as a process whereby researchers examine a specific subject in a thorough, detailed way. The subject could be an individual, a group, a community, a business, an organization, an event, or a phenomenon. The phenomenon was the concreteness fading which leaves many pupils unable to handle multiplication algorithms. In this design, groups of student teachers were classified based on their programme of study. The student teachers were given the Adinkra to learn and connect them to concrete materials. Kim (2020) used the case study design to develop sustainable teacher education and professional development by providing specific instructional strategies for conceptual understanding. Priya (2021) found that a case study is not just a method/technique/process of data collection but rather involves a detailed study of the concerned unit of analysis within its natural setting. This allows the researcher the leeway to use any method of data collection that suits their purpose (provided the method is feasible and ethical). The techniques of data collection used are questionnaire, survey, in-depth interview, participant/non-participant observation, and the study of documents, conversations in natural settings, signs, physical artifacts, and so on. So the use of Adinkra artifacts is novel and positively impacts the teaching and learning of mathematics.

Priya (2021) posits that a case study can either be explanatory, exploratory, or descriptive. In an Exploratory, the purpose here is to study a phenomenon to identify fresh research questions that can be extensively used in subsequent research studies. In Explanatory, the primary focus is to explain 'why' and 'how' certain conditions come into being. In a descriptive case study, the purpose is to 'describe' a phenomenon in detail in its real-world context. Priya (2021) (as quoted in Yin, 2014, p.7) used the case study design to describe the emergent subculture in an Italian slum in an urban neighbourhood in the United States, called the Cornerville district (a pseudonym).

The design enabled the researcher to describe in detail key phenomena such as the inter-subjectivity and interpersonal relations among the residents of the slum, the career advancement of the lower-income youth, and their ability (or inability) to break free from the neighborhood ties. In this study, the researcher adopted the design to help describe in detail the Adinkra in inter-subjectivity and interpersonal relations among the people, the knowledge advancement of the student teachers, and their abil-

ity to extend and transfer learning to pupils in the lower levels of education (Priya, 2021).

The Experimentations

Student teachers in this context, are students who are already teaching in the schools but have come to the university to be trained in professional skills and competencies to enable them to teach well. This is to boost not only their academic knowledge but also their pedagogical knowledge in the various subject areas. It is a form of continuous professional development training. Which is a requirement for every teacher to undertake to gain promotions and enhance their salaries and other conditions of services (Ministry of Education, 2020). In this cohort, 300 student teachers were admitted to learn professional pedagogies teaching and learning at the pretertiary levels of education. Since they all require the same pedagogical knowledge but the means of securing these Adinkra symbols were scarce, the researcher decided to use only 51 of them for this study.

Two stages of case study experimentation were undertaken with the 51 student teachers. These experimentations were based on two methods of teaching.

The two methods of teaching were the emergent and the surbonate. In the emergent, all the student teachers were tested after having explained the concept of the multiplication algorithms. This was called the 'pretest'. In the surbonate stage, the student teachers were given a set of tasks in the Adinkra symbols. In the Concrete stage, real objects were used to teach the multiplications. The researcher did not move away from these concrete examples when explaining the concepts. In the representation stage, objects were drawn or taken as snapshots. In the Abstract stage, no concrete examples were used and student teachers were taught using multiplication problems only (Kuepper-Tetzel, 2021).

In the surbonate stage, the researcher started with concrete objects, then moved to a paper-based version that increased the abstractness of the representation (Suh et al., 2020), but still used the objects from before, and lastly concluded with numeric representations alone. The multiplication problems used during the first phase were relatively easy (e.g. 3×12) as compared to the ones student-teachers had to solve during the abstract stage (e.g. 32×25) (Kuepper-Tetzel, 2021). After they had completed the tasks, they were given a set of tasks in multiplication of 1-digit by 2-digit numbers. Then the worksheets were retrieved and analyzed based on the stages of the Concreteness Fading principle (Febriana et al., 2019).

Populations and sample

Nearly 300 student teachers offered Post Diploma in Basic Education programmes at the University of Education, Winneba in Ghana (distance module). In this programme, student teachers can take their major elec-

tive areas in Integrated Science, English language, Social Studies, and Mathematics. In the mathematics options, there were about 60 student teachers. However, the researcher used the purposive sampling technique to select 51 student teachers who offered mathematics as their specialization. The 51 student teachers cut across genders, regions of Ghana, and knowledge of Ghanaian languages and cultures.

Purposive sampling is a technique used in qualitative research to select a specific group of individuals or units for analysis. Participants are chosen "on purpose," not randomly; it is judgmental or selective (Dovetail, 2023). Purposive sampling strategies move away from any random form of sampling and are strategies to make sure that specific kinds of cases that could be included are part of the final sample in the research study (Campbell et al., 2020). The reasons for adopting a purposive strategy are based on the assumption that, given the aims and objectives of the study, specific kinds of people may hold different and important views about the ideas and issues in question and therefore need to be included in the sample (Campbell et al., 2020)

There are many types of purposive sampling techniques, namely maximum variation, homogenous, typical case, extreme case, and critical case (Dovetail, 2023). Maximum variation sampling involves selecting a sample of individuals or units representing the maximum range of variation within the characteristics or attributes the researcher is interested in studying and is used to understand the widest possible diversity of experiences or viewpoints within the population (Dovetail, 2023). Homogeneous sampling involves selecting what is often a more narrow sample of individuals or units that are similar or have the same characteristics or attributes and is used to study a specific subgroup within the population in depth (Dovetail, 2023).

Extreme case sampling involves selecting a sample of individuals or units that are considered extreme or unusual in the characteristics or attributes the researcher is interested in studying and is used to understand unusual or exceptional experiences or characteristics within the population and are often viewed as outliers in a wider population (Dovetail, 2023). Critical case sampling involves selecting a sample of individuals or units that are important or central to the research question or the population being studied and is used to understand key experiences or characteristics within the population.

Expert sampling involves selecting a sample of individuals or units that have specialized knowledge or expertise in the topic or issue being studied and is used to gather insights and understanding from experts in the field, which can be used to develop follow-up studies (Dovetail, 2023). The researcher applied the typical case sampling to select the sample of individuals or units that were representative of the typical experiences or charac-

teristics of the population and is used to understand the most common or average experiences or characteristics within the population (Dovetail, 2023). In this sampling technique, the researcher satisfied the four most important principles clearly defining the purpose, and the sample was selected based on the characteristics or attributes, selecting a representative sample of the characteristics or attributes being studied, unbiased against anything other than the sample, and involving Expertise in the topic being. Without a solid understanding of the characteristics being selected, the population might not be as representative as it should be (Dovetail, 2023).

Instrumentations

The researcher used two sets of case study instruments to collect the data on the Adinkra. The first set of instruments was pretesting. In the pretesting, the researcher administered items based on the multiplication of one-digit by two-digit numbers via the conventional method of teaching. The second set of instruments was testing. In the testing, the researcher administered items based on “Concreteness Fading”, having taken the student teachers through the indigenous ‘Adinkra’ symbols. The items were similar to those administered in the pretesting (Ali, 2019). Biden (2022) contends that case study instruments focus on context and the depth of change, and, if done systematically, can generate bodies of data that can, in turn, be used for qualitative comparative analysis to analyze causation.

Data Analysis

The six most popular methods are Content analysis, Narrative analysis, Discourse analysis, Thematic analysis, Grounded theory, and Interpretive phenomenological analysis (Warren & Rautenbach, 2020). Content analysis is used to evaluate patterns within a piece of content (for example, words, phrases, or images) or across multiple pieces of content or sources of communication. For example, a collection of newspaper articles or political speeches (Warren & Rautenbach, 2020).

Narrative analysis is all about listening to people telling stories and analysing what that means. Since stories serve the functional purpose of helping us make sense of the world, we can gain insights into the ways that people deal with and make sense of reality by analysing their stories and the ways they’re told (Warren & Rautenbach, 2020). Discourse is simply a fancy word for written or spoken language or debate. So, discourse analysis is all about analysing language within its social context. In other words, analysing language — such as a conversation, a speech, etc — within the culture and society it takes place. For example, you could analyse how a janitor speaks to a CEO, or how politicians speak about terrorism (Warren & Rautenbach, 2020).

The thematic analysis looks at patterns of meaning in a data set — for example, a set of interviews or focus group

transcripts. But what exactly does that... mean? Well, a thematic analysis takes bodies of data (which are often quite large) and groups them according to similarities — in other words, themes. These themes help us make sense of the content and derive meaning from it.

Grounded theory is a powerful qualitative analysis method where the intention is to create a new theory (or theories) using the data at hand, through a series of “tests” and “revisions”. Strictly speaking, GT is more a research design type than an analysis method, but we’ve included it here as it’s often referred to as a method (Warren & Rautenbach, 2020). Interpretive Phenomenological Analysis (IPA) is designed to help you understand the personal experiences of a subject (for example, a person or group of people) concerning a major life event, an experience, or a situation. This event or experience is the “phenomenon” that makes up the “P” in IPA. Such phenomena may range from relatively common events — such as motherhood, or being involved in a car accident — to those which are extremely rare — for example, someone’s personal experience in a refugee camp. So, IPA is a great choice if your research involves analysing people’s personal experiences of something that happened to them (Warren & Rautenbach, 2020).

Phenomenology, developed by Edmund Husserl, is concerned with attending to the way things appear to individuals in experience, and aims at identifying the essential components of phenomena or experiences that make them unique or distinguishable from others. Followed up by Martin Heidegger, IPA attempts to understand what it is like to stand in the shoes of a subject (although recognising this is never completely possible) and through interpretative activity make meaning comprehensible by translating it (just like the mythological Hermes translated the gods’ messages to humans). Thus, IPA synthesizes ideas from phenomenology and hermeneutics resulting in a descriptive method because it is concerned with how things appear and letting things speak for themselves, and an interpretative because it recognizes there is no such thing as an uninterpreted phenomenon (Pietkiewicz & Smith, 2012). Ismail and Kinchin (2023) have enumerated four main different approaches to phenomenology.

- Existential Phenomenological Analysis (EPA)
- Hermeneutic Phenomenological Analysis (HPA)
- Interpretative Phenomenological Analysis (IPA)
- Transcendental Phenomenological Analysis (TPA)

Each of these four types offers a unique perspective on understanding the lived experiences of individuals and their meanings. However, the practice epoché, also known as phenomenological reduction, is recommended to help one suspend one’s preconceptions and biases but rather observe and describe phenomena as they appear to the consciousness and to attend to their essential qualities (Ismail & Kinchin, 2023).

As a process, IPA coding started with a process of ‘initial commenting’ or ‘initial noting,’ in which the researcher

wrote his initial analytic observations or brief commentaries about the data. Then, the researcher coded his first data items from the pretest and then progressed to developing themes for that data item. So the researcher coded each stage of the analysis for each data item, before moving to the next. In terms of procedures for theme development, there were two levels of themed development in IPA referred to as ‘emergent’ and superordinate themes. Emergent themes were noted on the multiplication data items, while superordinate themes (Adinkra) were developed from emergent themes. Once coding and theme development were complete for each dataset, the researcher developed superordinate themes across the dataset to end with an organizing framework for the analysis as seen in Figures 2–5 (Ismail & Kinchin, 2023).

Validity and Reliability

The researcher chose test-retest reliability to assess whether the performance in the emergent stage yielded the same results as the performance in the “surbonate stage. Indeed, the Cronbach alpha coefficient yielded 0.78 (Bhattacharjee, 2021). In validity, the researcher assessed three main measures of validity, vis-à-vis criterion, construct, and content. This measured all the student teachers’ knowledge in “Concreteness Fading”.

Threats to Internal validity

The researcher controlled history (unrelated events such as fatigue that nearly influenced the performance), maturation (outcomes of the performance in the post-test that were nearly influenced by the pre-test), instrumentation (differences in times of the pre-test and post-test that nearly influenced performance) and testing (pre-test that nearly influenced the post-test). These were accomplished by manipulation, elimination, inclusion, statistical control, and randomization (Bhattacharjee, 2021).

Threats to Internal Validity were accomplished by manipulation, elimination, inclusion, statistical control, and randomization (Bhattacharjee, 2021). In manipulation, the researcher manipulated the independent variables in one or more levels (called “treatments”) and compared the effects of the treatments against a control group where subjects did not receive the treatment. In elimination, the researcher eliminated extraneous variables by holding them constant across treatments and by restricting the study to only student teachers who specialised in mathematics. In inclusion, the researcher considered the role of extraneous variables by including them in the research design and separately estimating their effects on the dependent variable to allow for greater generalizability. In statistical control, the researcher measured the extraneous variables and used them as covariates during the statistical testing process. In randomization, the researcher canceled out the effects of extraneous variables through a process of a random sampling of the distance module student teachers to be assured that any effects were of a random (non-systematic) nature. The 51 stu-

dent teachers were selected randomly from the 300, and randomly assigned to treatment groups (Bhandari, 2022).

Ethical Considerations

The researcher paid attention to voluntary participation, informed consent, anonymity, confidentiality, potential for harm, and results communication (Ali, 2022). Ethical protection aims to protect the rights of research participants, enhance research validity, and maintain scientific or academic integrity (Ali, 2019).

The purposive sampling technique requires the researcher to ensure that the study is conducted ethically and that the rights of the participants are protected. This may require obtaining informed consent from the individuals in the sample and safeguarding their privacy (Dovetail, 2023). The researcher followed several ethics procedures before the administration of the research instruments. Before the researcher embarked on the data collection, permission was granted by the Coordinator and the Administrator of the Distance Learning Unit. This ensured that research participants were adequately protected. It also ensured that the researcher carried out the project at a minimal cost (Ali, 2019).

During the data collection time, it was recommended that the researcher should feel comfortable with moments of silence, to allow both oneself and the participant to reflect on issues being discussed. An experienced interviewer must also be sensitive to and try to be aware of all verbal, nonverbal, and non-behavioural communication. For ethical reasons, and because IPA studies are frequently concerned with significant existential issues, the researcher must monitor how the process is affecting the participant to determine when the participants avoid talking about certain issues, start feeling awkward, ashamed, or become very emotional. The duration of most IPA data collection is one hour or longer. In IPA it is necessary to audio record the respondents' voices and produce a verbatim transcription (Pietkiewicz & Smith, 2012). The participants also consented to the research work. This ensured that the research participants' rights were protected. This was also to hold the researcher liable for research breaches against informed consent, anonymity, confidentiality, privacy, plagiarism, and fraud. The researcher also informed student teachers of the intended dissemination and publications of the study in workshops, seminars, conferences, and journal publications (Ali, 2019).

Results and Discussion

Research Question 1: Task 1: "Adinkra" as "Friendship"

What value does 'friendship' bring to the multiplication of numbers? "Friendship in Adinkra is found in the heart symbol. The heart is the center of your thinking and acceptance of people you may not know but you want to gain knowledge from them".

As seen in Figure 1, it was clear

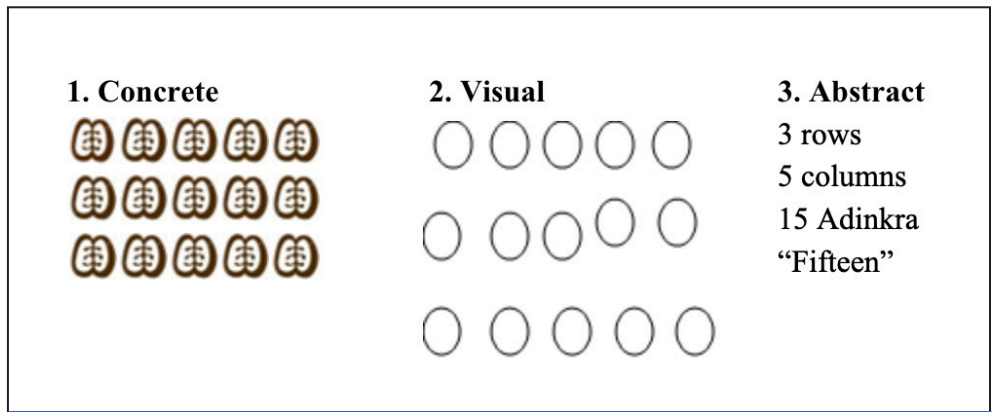
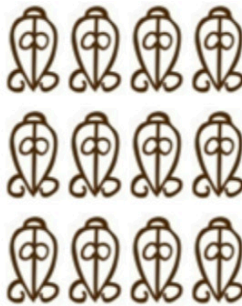


Figure 2. Concrete to Visual transition (Credit: Pearce & Orr, 2018)

1. Concrete



2. Visual



3. Abstract

$$3 \times 4 = 12 = (10 + 2)$$

Figure 3. Linear Concrete to Visual transition (Credit: Pearce & Orr, 2018)

that the student-teacher brainstormed the issues around friendship and related the concept to learning. Dicken (2022) says that one may think a friendship will last forever but it is not uncommon for some friends to fade. Sometimes, a disagreement or falling out creates a gap between friends. Other times, commitments like work, distance, or family result in a friendship slowly fading away without animosity. Student teachers accepted the value of 'friendship' and believed that learning multiplication starts with real friendship. They applied the value during the group discussions in a congenial and cordial atmosphere. Concrete to Visual transition offered solutions for making these connections as in Figure 2.

In Figure 2, Pearce and Orr (2018) started with these concrete manipulatives, progressed to drawing those representations, and finally, represented the mathematical thinking abstractly through symbolic notations. "Adinkra" are varied symbols and represent various mathematical contexts (Ali, 2018). The "Adinkra" symbols represent concepts or aphorisms and are used extensively in fabrics, pottery, logos, and advertising (Efiabevi, 2013). Even though one cannot directly link them to numbers and operations, they abound in rich affective attitudes (strength, forgiveness, faithfulness, friendship, and peace) and values (e.g. royalty, truth, courage, and understanding) enshrined in the new Ghanaian curricula of pre-tertiary education levels (Ali, Davis, & Agyei, 2021; Ministry of Education, 2019).

Research Question 2: Task 2: "Adinkra" as "Power of Love"

How does the value of 'love' help in the multiplication of numbers? "Mathematics is a difficult subject. So if you want to help you solve problems then there must be someone who loves the subject. If the person does not love the subject they will not be able to solve the problems. So love brings power!"

As seen in Figure 1, the Adinkra symbol for love is designed to connect patterns and signs. As a wonderful design, it brings learners together to strengthen their collective effort. The power of love for mathematics has widely been discussed. For instance, Brown (2019) opines that the four ways to add love for mathematics are to commit to inspiring teaching, preach the value of mistakes, advocate for growth mindsets, and strengthen your mathematics skills. These four ways were adequately exemplified in carrying out the activities with the "Adinkra" symbol in the "Concrete Fading" as shown in Figure 4. Figure 3 shows how the conceptual understanding of "Love" continues to deepen through the use of the "Power of Love". At this stage, student teachers continued sharing using visuals and gradually introducing symbolic notations. Different representations of concrete materials were required to consolidate learning. Moreover, since student

teachers had a significant amount of time to inquire, investigate, and solve problems using both concrete and visual representations, they readily developed the ability to visualize different representations in their minds. It was more efficient to use symbolic notations and operations in multiplication rather than building concretely or drawing visually (Pearce & Orr, 2018).

Research Question Three: Task 3: “Adinkra” as “Strength”

Why does the value of ‘strength’ help in the multiplication of numbers? “Getting strengthened to solve multiplication is a great motivation. It is sometimes frustrating to solve multiplication problems that have not been connected to learning resources.”

The value of ‘strengthen’ is seen in Figure 1 four interwoven and intertwined circular symbols that show that ‘unity is strengthen’. One person cannot withstand the challenges of confronting complex mathematics tasks. Bowen (2021) has acknowledged a strength-based perspective of learning mathematics and advocated for lessons from a strengths-based perspective. This means focusing on what students already know, uncovering their strengths, and building on those strengths through instruction. This view is an excellent way to start using symbols from student teachers’ environment culture and customs and scaffold them to much higher heights (Ali, Davis, & Agyei, 2021).

Figure 4 shows how “Strength” propelled the student teachers to adequately prepare and enter into the abstract stage. As student teachers were shown how easy multiplication can be performed by having a conceptual understanding, they eventually jumped on the opportunity to multiply numbers without using manipulatives and representations. The more students linked their concrete and visual models to more abstract representations, the stronger their conceptual understanding supported any procedural approaches to progressive and effective algorithms in multiplication (Pearce, & Orr, 2018).

Research question four: Task 4: “Adinkra” as “Intelligence”

For whom does the value of ‘intelligence’ help in the multiplication of numbers? “Intelligence is the major driver is mathematics learning and achievement. Without intelligence, you cannot pursue mathematics at any level. So, this value encourages us to work harder and attain higher achievements in mathematics.”

This value in Figure 1 has the four cardinal points in an oval. The cardinal points are an indication that intelligence is multifaceted and multi-directional. So, the value of intelligence helps learners who are hardworking, multi-task, and all-round good in computational reason-

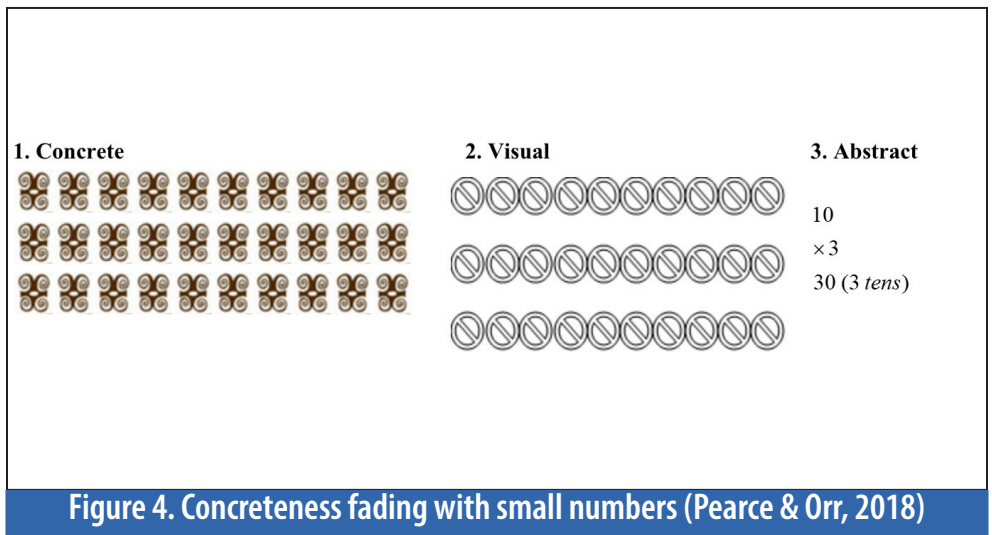


Figure 4. Concreteness fading with small numbers (Pearce & Orr, 2018)

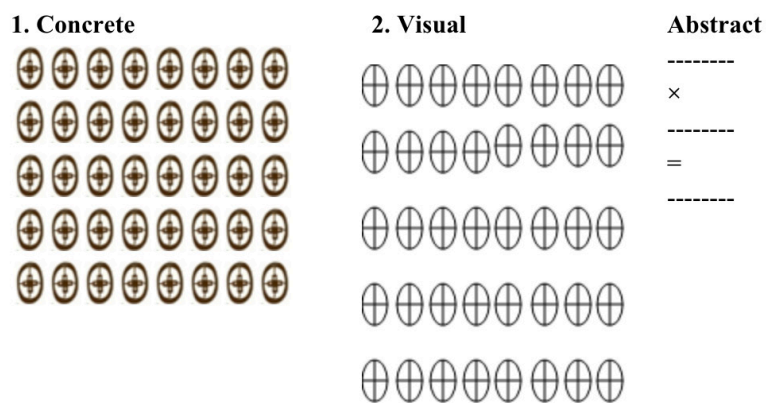


Figure 5. Concreteness fading with scaffolding activity (Pearce & Orr, 2018)

ing. Logsdon (2022) found that children with increased logical-mathematical intelligence are typically methodological and logical in thinking. Children may be adept at solving mathematics problems in their heads and drawn to logic puzzles and games. Mathematics “intelligence” exhibits children’s abstract concepts, categorization, classification, patterns, problem-solving, and visual analysis (Bartolomei-Torres, et al., 2022).

In Figure 5, student teachers have fully applied intelligence to transition and transfer knowledge into the abstract stage. In this stage, they scaffolded the concreteness fading models; they could easily conceptualize the movement without being assisted (Ali & Anderson, 2021).

Discussion

As originally conceived, it is a three-step progression that begins with enacting a physical instantiation of a concept, moves to an iconic depiction, and then fades into a more abstract representation of the same concept (Suh et al., 2020). It takes its source from Bruner’s iconic, enactive, and symbolic stages (Bruner, 1966). This shows that learning from concrete materials that “fade” to abstract symbols benefits transfer, the progression from concrete to

abstract is better than the reverse, learning from concrete materials is similar to learning from abstract symbols, and the benefits of “fading” extend to children with low and high prior knowledge. Ching and Wu (2019) examined the effectiveness of various instructional strategies that aimed to enhance children’s understanding of the inversion concept using 140 kindergarten pupils randomly assigned to each group of concrete-only, abstract-only, concreteness fading, abstract- to-concrete, and control. All the intervention (experimental) groups showed significantly greater progress than the control group in solving the inversion problems in the post-tests. In their findings, it was revealed that concrete representations were more effective than abstract representations. The superior benefits of concreteness fading appeared more prominent in the post-test scores for children with lower prior knowledge. The findings of Ching and Wu (2019) brought to light two key implications:

- Concrete representations should not be avoided in teaching mathematics
- The order of the various representations is key for effective learning.

In particular, Pearce and Orr (2018) made significant and

inspirational findings on the “Concreteness Fading” of the multiplication of numbers. Pearce and Orr (2018) discovered that using circular manipulatives like doughnuts was more concrete than drawing doughnuts or using symbols (numbers and operations). It was revealed that using concrete manipulatives was still more abstract than using the actual items in the quantity being measured.

On average, student teachers performed more problems correctly using “Concreteness Fading”. Kuepper-Tetzl (2021) discovered that heavy use of concrete objects and examples without abstracting from them can be detrimental to solving mathematics problems. “Concreteness Fading” can also always be done in different ways, namely providing concrete examples first, then substituting concrete with more abstract ones, and, finally, moving completely to an understanding of the abstract principles (Kuepper-Tetzl, 2021). This has completely answered and satisfied the domain of the study.

Conclusion

The findings show that student teachers could readily navigate easily from the concrete into the abstract stage in the multiplication of 1-digit by 2-digit numbers. Particularly, it was evident that the student teachers had a lot of fun and play when using the “Adinkra” symbols. The findings also show that the student teachers improved in the “Concrete Fading” tasks as compared to the tasks involving the usual conventional methods of multiplication. The improvement was much more remarkable in the transition from visual to abstract than from concrete to visual. This big improvement was attributable to the “Concreteness Fading” principle and it is “Adinkra” that added more impetus to the tasks. This is contrary to the expectation that real concrete materials enhance learning than abstract ones. It is rather the value and virtue of the concrete material that matters more!

We, therefore, recommend that student teachers must always avoid rushing to symbols and symbolic manipulations of mathematics. It is not uncommon to observe many student teachers rushing towards using symbols for the multiplication of 1-digit by 2-digit numbers. This tends to veer the classroom into instrumental learning without making any deeper understanding and appreciation of the concepts therein.

We also recommend that student teachers must endeavour to help their children develop a firm grasp of their indigenous artefacts like “Adinkra” in every mathematics domain. Even though the curriculum is emphatic on the use of local and indigenous materials that are easily accessible to pupils, it is not uncommon to hear many teachers complain about a lack of teaching and learning resources. Materials are readily available for addition and subtraction. However, resources to use for multiplication activities in school mathematics remain untapped.

References

- Aduko, E. A., & Armah, R. B. (2022). Adapting Bruner’s 3-tier theory to improve teacher trainees’ conceptual knowledge for teaching integers at the basic school. *European Journal*
- Ali, C. A. (2022). The didactical phenomenology in learning the circle equation. *International Electronic Journal of Mathematics Education*, 17(4), 1–11 <https://doi.org/10.29333/iejme/12472>.
- Ali, C. A. (2021). Ghanaian Indigenous Conception of Real Mathematics Education in Teaching and Learning of Mathematics. *Indonesia Journal of Science, Technology, Engineering and Mathematics (IJSTEM)*, 4(1), 82–93. <https://doi.org/10.24042/ijstme.v4i1.7382>
- Ali, C. A., Davis, E. K., & Agyei, D. D. (2021). Effectiveness of semiosis for solving the quadratic equation. *European Journal of Mathematics and Science Education*, 2(1), 13–21. <https://doi.org/10.12973/ejmse.2.1.13>.
- Ali, C. A., & Anderson, H. K. (2021). Pre-Service Teachers’ Pedagogical Content Knowledge in Transferring From Basic Musical Notations to Basic Fractions. *Journal of STEM Education: Innovations and Research (JSTEM)*, 46–50.
- Ali, C. A. (2019). Didactical conceptual structures in extending the triad to the tetrahedron exemplified in the teaching and learning of equations of the circle. PhD Thesis, Faculty of Science and Technology Education, University of Cape Coast, Ghana
- Babbitt, W., Lachney, M., Bulley, E., & Eglash, R. (2015). Adinkra Mathematics: A study of ethnocomputing in Ghana. *Multidisciplinary Journal of Educational Research*, 5(2), 110–135, doi:10.17583/remie.2015.1399.
- Bartolomei-Torres, et al. (2022). Logical-mathematical intelligence: definition, characteristics, and activities for its development, Our Team, Retrieved from <https://www.learningbp.com/logical-mathematical-intelligence-definition-characteristics-activities-development/>.
- Bhandari, P. (2022). Internal validity in research | definition, threats & examples. Scribbr. Retrieved on February 20, 2023, from <https://www.scribbr.com/methodology/internal-validity/>.
- Biden, A. (2022). 10 Examples of Qualitative Data. Tools4dev., retrieved from <https://tools4dev.org/blog/examples-of-qualitative-data/>
- Bhattacharjee, A. (2021). Improving internal and external validity. University of South Florida via Global Text Project, Retrieved from <https://socialsci.libretexts.org/Bookshelves/>.
- Boddy-Evans, E. (2020). The Origin and Meaning of Adinkra Symbols. Springer Nature, retrieved from <https://www.thoughtco.com/origin-and-meaning-of-adinkra-symbols-4058700>.
- Bowen, J. (2021). Ask the Expert: How can teaching math from a strengths-based perspective help students succeed? College of Education News, Retrieved from <https://ced.ncsu.edu/news/2021/06/17/ask-the-expert-how-can-teaching-math-from-a-strengths-based-perspective/>.
- Brown, K. S. (2019). 4 ways to add up a love for math, e-School News, Retrieved from <https://www.eschoolnews.com/featured/2019/02/20/love-math/>.
- Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge: Belknap.
- Campbell, S., Greenwood, M., Prior, S., Shearer, T., Walkem, K., Young, S., Bywaters, D., & Walker, K. (2020). Purposive sampling: complex or simple? Research case examples. *Journal of Research in Nursing*, 25(8): 652–661. doi: 10.1177/1744987120927206.
- Ching, B. H. H., & Wu, X. (2019). Concreteness fading fosters children’s understanding of the inversion concept in addition and subtraction. *Learning and Instruction*, 61(1), 148–159. Elsevier Ltd. Retrieved from <https://www.learnlib.org/p/208159/>.
- Dicken, L. (2022). How to deal with a fading friendship, Retrieved from <https://www.wikihow.com/Deal-With-a-Fading-Friendship>
- Donovan, A. M., & Fyfe, E. R. (2022). Connecting concrete objects and abstract symbols promotes children’s place value knowledge, *Educational Psychology*, 42 (8), 1008–
- Douglas, H., Headley, M. G., Haddenc, S., & LeFevrea, J-A. (2020). Knowledge of mathematical symbols goes beyond numbers. *Journal of Numerical Cognition*, 6(3), 322–354, <https://doi.org/10.5964/jnc.v6i3.293> Received: 2020-03-27.
- Dovetail (2023). What is purposive sampling? Dovetail Editorial Team, retrieved from <https://dovetail.com/research/purposive-sampling>
- Efiabevi (2013). Adinkra. Freedom Hall. Retrieved from https://freedomhallblog.wordpress.com/2013/04/30/524746_525071414223355_293027976_n-jpg/
- Febriana, D. F., Amin, S. M., & Wijayanti, P. (2019). Concreteness fading process of elementary school students based on mathematical ability. *IOP Conf. Series: Journal of Physics: Conf. Series* 1157-042049.
- Fyfe, E. R., & Nathan, M. J. (2019). Making “concreteness fading” more concrete asa theory of instruction for promoting transfer. *Educational Review*, 71(4), 403–422.
- Ismail, N. S., & Kinchin, G. D. (2023). The Construct of Phenomenological Analysis: A Case Study of Interpretive Phenomenological Analysis (IPA). *The Scientific Journal of Egypt*, 1 (1), 7–17. <https://doi.org/10.52649/egscj230809>.

- Jross (2023). When and How to Use a Case Study for Research. Copy press Writer, retrieved from <https://www.copypress.com/kb/measurement/when-and-how-to-use-a-case-study-for-research/>
- Kim, H. J. (2020). Concreteness fading strategy: a promising and sustainable instructional model in mathematics classrooms, *Sustainability*, 12(6), 2211, <https://doi.org/10.3390/su12062211>
- Kokkonen, T., Lichtenberger, A., & Schalk, L. (2022). Concreteness fading in learning secondary school physics concepts, *Learning and Instruction*, 77(1), 1-11, Elsevier Ltd, <https://doi.org/10.1016/j.learninstruc.2021.101524>.
- Kokkonen, T., & Schalk, L. (2020). One instructional sequence fits all? a conceptual analysis of the applicability of concreteness fading in mathematics, Physics, Chemistry, and Biology Education. *Education Psychology Review*, DOI: <https://doi.org/10.1007/s10648-020-09581-7>.
- Kuepper-Tetzl, C. (2021). Concreteness fading: a method to achieve transfer. learning scientists posts, for teacher, retrieved from <https://www.learningscientists.org/blog/2018/2/1-1>.
- Kuwornu-Adjaottor, J. E. T., Appiah, G., & Nartey, M. (2015). The philosophy behind some Adinkra symbols and their communicative values in Akan. *Philosophical Papers and Review*, 7(3), 22-33, <https://doi.org/10.5897/PPR2015.0117>
- Logsdon, A. (2022). Logical-mathematical learning style, very well family, Retrieved from <https://www.verywellfamily.com/mathematical-logical-learners/>
- McNeil, N. M., & Fyfe, E. R. (2012). "Concreteness fading" promotes transfer of mathematics knowledge. *Learning and Instruction*, 22(6), 440-448. DOI: <https://doi.org/10.1016/j.learninstruc.2012.05.001>.
- Ministry of Education (2019). New Curriculum for Upper Primary (Basic 4-6). Accra: National Council for Curriculum and Assessment (NaCCA).
- Okyere, M. (2021). Culturally responsive teaching through the Adinkra symbols of Ghana and its impact on students' mathematics proficiency. Doctor of Philosophy Thesis, University of Alberta, retrieved from 1c3c666c-e9cf-48b5-9a45-12510894af15.pdf.
- Pearce, K., & Orr, J. (2018). Make mathematics matter with concrete fading. Retrieved from <https://tapintoteenminds.com/concreteness-fading/>.
- Pickering, N. (2023). Concrete representational abstract sequence. manoeuvring the middle school, retrieve from maneuveringthemiddle.com/difficult-math-concepts/
- Pietkiewicz, I. & Smith, J.A. (2012). Praktyczny przewodnik interpretacyjnej analizy fenomenologicznej w badaniach jakościowych w psychologii. *Czasopismo Psychologiczne*, 18(2), 361-369.
- Priya, A. (2021). Case Study Methodology of Qualitative Research: Key Attributes and Navigating the Conundrums in Its Application. *Sociological Bulletin*, 70(1), 94-110. <https://doi.org/10.1177/0038022920970318>
- Suh, S., Lee, M., & Law, E. (2020). How Do We Design for Concreteness Fading? IDC '20, June 21-24, 2020, London, United Kingdom.
- Warren, K., & Rautenbach, E. (2020). Qualitative Data Analysis Methods 101: The "Big 6" Methods + Examples. GradCoach, retrieved from <https://gradcoach.com/qualitative-data-analysis-methods/>

Professor Clement Ayarebilla Ali (Ph.D.) is an Associate Professor in Mathematics Education in the Department of Basic Education, University of Education, Winneba. He has over 20 teaching experience in teacher education, training, and research in Ghana. Clement has published over 50 articles and books, presented over 40 peer-reviewed conference papers, and written mathematics modules for distance and e-learning. He has reviewed theses, books, articles, and conference papers for internationally renowned publishers. His research interest is in mathematics education, didactics of mathematics, teacher education, early childhood education, and psychology of mathematics education. He is married and blessed with three children.

